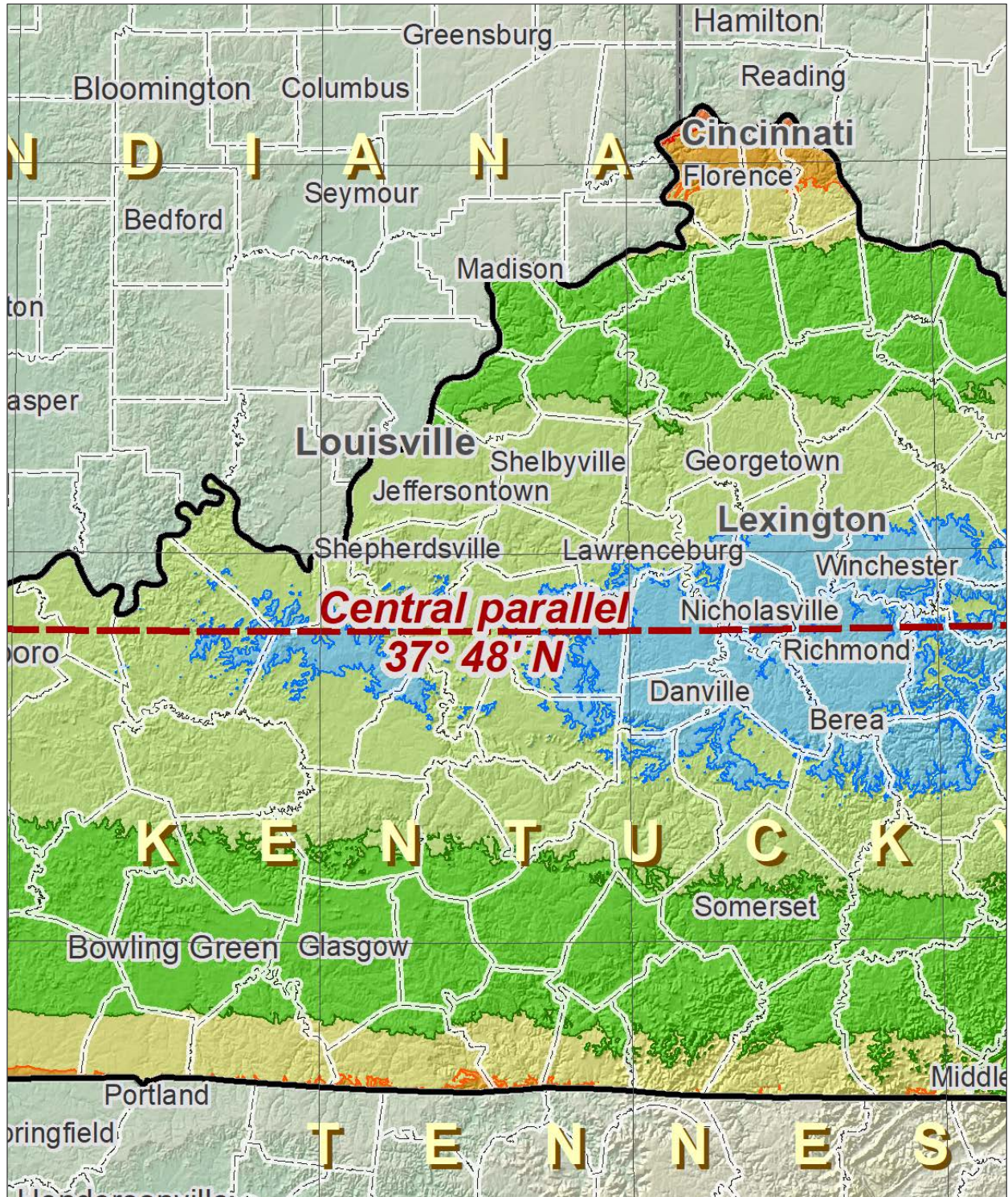


# The Kentucky State Plane Coordinate System *Standards and Specifications Document*



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# **The Kentucky State Plane Coordinate System**

## ***Standards and Specifications Document***

### **Introduction, Purpose, and Scope**

In anticipation and preparation for the eventual adoption of the 2022 Terrestrial Reference Frames (2022 TRFs) and the North American-Pacific Geopotential Datum of 2022 (NAPGD2022) by the National Geodetic Survey (NGS), the Commonwealth of Kentucky has embarked on an effort to reformulate its implementation of the national State Plane Coordinate System (SPCS). In past iterations it has been customary for states to adopt their implementation of the SPCS as static components of the National Spatial Reference System (NSRS) and therefore be completely defined within the statutory and/or regulatory language on the assumption that changes would rarely occur, if ever. In 1992 Kentucky adopted the Kentucky Plane Coordinate System of 1983 through the passage of KRS 1:020 and in 2001 the Kentucky Single Zone Coordinate System of 1983 (KY1Z83) through the promulgation of 10 KAR 5:010, both of which statically defined their relevant parts of Kentucky's overall SPCS within the statutory and regulatory language. This approach, while reasonable at the time, will now require states with SPCS legislation on their books to revise their relevant statutes through the legislative and regulatory processes once the planned new datums are established and adopted at the national level. Kentucky is no exception in this regard.

While this approach has worked well in the past it is becoming increasingly evident that a more dynamic and comprehensive strategy for defining, managing, and maintaining the SPCS across various datums at the state level is warranted, particularly with the advent of time dependency, through epochs, having been introduced into the current NSRS vernacular, and the relatively recent development of low distortion coordinate reference systems (LDCRS) having been adopted outside the purview of the national SPCS by various states and tribal authorities across the country. Time dependency aside, these LDCRSs have been primarily established in an effort to address shortcomings of the existing SPCS due to the original design objective of accounting for distortions experienced on the projection surface relative to the defining ellipsoid as opposed to the topographic surface covered by a given zone, with the latter approach being far superior as a practical matter.

This document represents the culmination of Kentucky's effort to address the dynamic nature of how state plane coordinates are currently experienced through the various realizations and epochal adjustments of the North American Datum of 1983 (NAD 83) and through future versions of the NSRS as signaled by policy changes recently adopted by NGS. While the Kentucky State Plane Coordinate System (KSPCS) will continue to be established and adopted through the statutory and regulatory processes (legislation and promulgation), the fine-grained details of how the system will be defined on various datums through the concept of series, layers, and zones will be addressed within this document.

The scope of this document is to provide a holistic and reasonably comprehensive discussion of the KSPCS as it pertains to its origins and historical implementations, from a practical surveying perspective, as well as technical aspects such as the use of well-established

conformal projection methods and techniques required to achieve low distortion results when desired and optimum performance for large areas such as statewide single zone coverage. Also included in this scope will be a discussion on conformance with NGS policy and standards such as meter to customary foot conversion factors. Finally, this document will present the defining parameters established and adopted for all SPCS zones as they apply to the datums upon which they are based, meaning all datums past and present will be covered, not just the datum currently in effect. For the purposes of this document the term *customary foot* will be utilized when referencing non-metric linear units of measure outside the context of a specific meter-to-foot conversion factor.

## Acknowledgements

This document represents the culmination of a concerted effort made possible through the contributions of individuals who all share a common interest in supporting geospatial activities throughout the Commonwealth of Kentucky and beyond its borders, particularly given the national importance of the State Plane Coordinate System (SPCS). In particular, the following people are acknowledged for providing their professional talents and technical insights to this project:

**Michael Dennis**, who's prolific work has set the stage for modernization of the SPCS through his development of specialized techniques for applying low distortion mapping projections on a regional scale, and his current management of the National Geodetic Survey's SPCS modernization project for the four Terrestrial Reference Frames of 2022 (TRF2022) deserves special thanks for taking valuable time from his busy schedule to foster, advise, review, and contribute to this important project in spite of a heavy workload, looming deadlines, and commitments related to NGS TRF2022 activities.

**David Doyle** and **Renee Shields-Doyle**, both of the National Geodetic Survey (*Ret*) and participants in the original North American Datum of 1983 (NAD 83) Team have graciously provided their unique knowledge and expertise in this particular branch of geodesy. David is recognized throughout the Commonwealth as a top emissary from NGS on matters geodetic, and over the past three decades has enlightened at least two generations of surveyors and various geospatial professionals on the *black art* of geodesy in general, and specifically the inner workings of the various North American datums, particularly NAD 83. His advisory role in the development of the original *Kentucky Single Zone Coordinate System of 1983* and contributions within this document provides a unique voice of eminent technical and historical value.

**Ross Mackay**, former NGS Advisor to Kentucky, and **Jeff Jalbrzikowski**, current NGS Regional Advisor to Kentucky have provided valuable review and advisory roles in the development of this document. In addition to this particular project, Ross MacKay's personal contributions during his tenure as NGS Advisor to Kentucky brought about vast improvements regarding the state of geodesy in the Commonwealth, particularly in the development of the CORS and Federal Base Network (FBN) programs. **William Stone**, NGS Southwest Region Geodetic Advisor, provided technical review and insights in general, but more specifically regarding the planned terrestrial reference frames for 2022. His contribution has resulted in a more concise narrative on that topic.

**Kent Anness**, Branch Manager at the Kentucky Division of Geographic Information, has provided invaluable management skills in keeping subcommittee meetings on track and on topic as well as assisting **Mike Sunseri** of the Kentucky Office of Homeland Security, who fostered the latest revisions to *KRS Chapter 1:010 and 1:020* successfully through the legislative process. Kent has also been instrumental in implementing revisions to 200 KAR 041:010 through the administrative regulation process, not to mention his skillful outlining and formatting of this document has been a source of relief to the author.

**Perry Semones**, Survey Manager at the Kentucky Transportation Cabinet (KYTC), through various interviews and conversations provided relevant historical insights as well as digging out several historical documents of special relevance from within the KYTC's archives, some of which were probably retrieved from under his desk.

Participants in the Kentucky Geographic Information Advisory Council (GIAC) KSPCS subcommittee not mentioned above include **Bill Haneberg**, State Geologist and Director, Kentucky Geological Survey; **James E. Manning**, Director, Kentucky Board of Licensure for Engineers and Land Surveyors; **Kim Anness**, Kentucky Division of Geographic Information; **Will Holmes**, GIS Manager, Kentucky Transportation Cabinet; **Danielle Kelly**, KyCORS Manager, Kentucky Transportation Cabinet; **Timothy Tong**, Kentucky Association of Professional Surveyors (KAPS); **Steve Chino**, Kentucky Association of Mapping Professionals (KAMP); **Ron Householder**, KAMP.

## Authority

This document has been mandated through the revision of KRS Chapter 1:020 and subsequent promulgation of 200 KAR 041:010, Section 2 in which:

- (1) The Commonwealth Office of Technology (COT), as advised by the Geographic Information Advisory Council (GIAC), shall develop and maintain the *KSPCS Standards and Specifications Document*.
- (2) The *KSPCS Standards and Specifications Document* is incorporated by reference.

The general requirements for incorporation by reference as specified in KRS 13A:224 are met as follows:

- (1) The material incorporated by reference relates only to the specific subject matter pertaining to the KSPCS.
- (2) The material has been reviewed in detail by the Commonwealth Office of Technology as advised and formally adopted by the Kentucky Geographic Information Advisory Council (GIAC).
- (3) No state statute or federal law prescribes the same or similar procedure, or sets forth a comprehensive scheme of regulation on the subject matter.
- (4) This incorporation is necessary in order to establish and describe practice and procedures for implementation of the KSPCS as authorized by KRS 1:020(2).



## **Living Document**

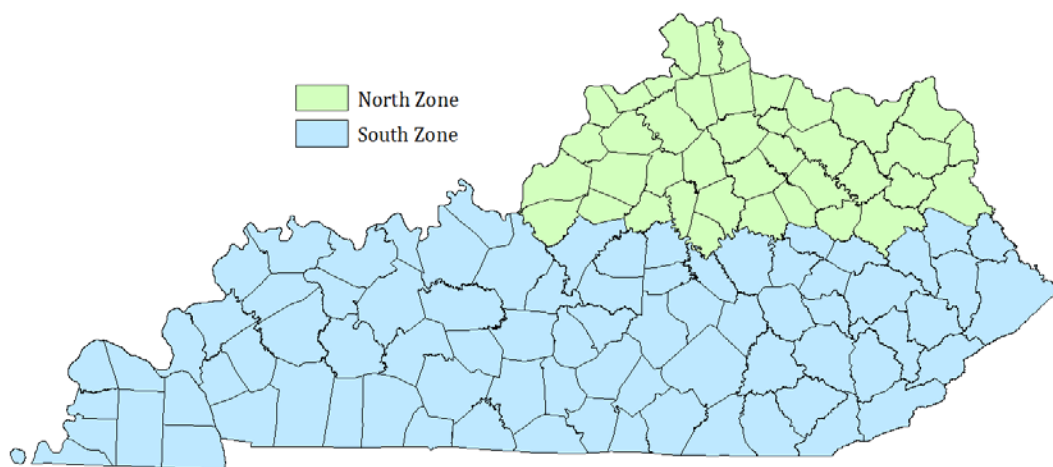
This document has been designed and formulated to be a living document and will be revised as required to keep up with changes to policies and standards at the state and federal levels, including the establishment and adoption of new datums and reference systems currently under development by the National Geodetic Survey. Minor revisions and corrections may also occur as they are deemed necessary and/or appropriate.

## Chapter 1: History

The history of the SPCS is well documented, most recently and rather comprehensively by NGS through the publication of *NOAA Special Publication NOS NGS 13, The State Plane Coordinate System - History, Policy, and Future Directions* (Dennis, 2018), along with *NOAA Manual NOS NGS 5, State Plane Coordinate System of 1983* (Stem, 1990). While this document intends to treat this topic in a general sense on the national level while providing more detailed insights with respect to Kentucky's implementation, particularly from the perspective of general surveying practices in use during their respective times, the reader is directed to the above referenced documents to gain a detailed look into the evolution of the SPCS and underlying datums, including the usage of various mapping projections in general prior to its adoption.

### 1.1 - North American Datum of 1927 (NAD 27)

The national State Plane Coordinate System originated through a request from the North Carolina State Highway and Public Works Commission in 1933 to the then United States Coast and Geodetic Survey (USC&GS), now NGS, for the creation of a system of geodetically referenced plane coordinates for the state, presumably based on the then newly established North American Datum of 1927 (NAD 27). Upon developing a statewide system for North Carolina using the Lambert Conformal Conic (LCC) projection and subsequently developing a statewide system for New Jersey using the Transverse Mercator (TM) projection, USC&GS decided to aggressively expand this concept nationally by creating a system of state plane coordinates for all 48 states in the Union at that time. In 1934 such a system, named the State Plane Coordinate System of 1927 (SPCS27), had been completed and was comprised of a total of 110 zones covered by 66 LCC projections (including 2 for Kentucky, North Zone and South Zone respectively) and 44 TM projections. This system was based on the Clarke Ellipsoid of 1866, which for NAD 27 was fixed and oriented to the North American continent, and utilized a meter-to-foot conversion factor for defining linear units of measure identical to the U.S. survey foot (a name that was assigned in 1959).



**Figure 1.1.1:** KSPCS North and South zone coverage areas as defined on NAD 27 and NAD 83.

During the subsequent decades following the establishment of SPCS27 the system was actively promoted, documented, and updated as new zones were added, most notably in 1960 after Alaska and Hawaii were granted statehood in 1959, and later on when zones for various U.S. territories were added to the system. By 1956 at least 24 states had adopted SPCS27 through legislative enactment and by the late 1980s 18 additional states had followed suit for a total of 42 states having SPCS legislation, with Kentucky being one of the few exceptions.

That is not to say Kentucky's portion of SPCS27 had not been implemented at the project level. In 1956 during preparation for the design and construction of Kentucky's portion of the Interstate Highway System the Kentucky Transportation Cabinet published a manual titled *The Plane Coordinate System in Kentucky - Manual for Highway Engineers* in which the Department of Highways required select projects to reference all horizontal measurements to the State Plane Coordinate System. The 1956 Kentucky SPCS manual also references an extensive statewide network of horizontal and vertical control monuments jointly established by the United States Geological Survey (USGS) and USC&GS to facilitate the completion of topographic mapping projects (presumably USGS 7.5 minute quadrangle topographic map series) using aerial surveying methods. It was from this control network that local control for future highway projects, particularly those associated with the Interstate Highway System, would be established using SPCS27.

In addition to highway projects, the Tennessee Valley Authority (TVA) had also adopted SPCS27 across several states, including Kentucky, for establishing geodetic control for its various projects and assets (TVA, 1951). Also, with continued utilization of aerial surveying methods for various large-scale mapping projects, such as utility and cadastral mapping, and local control networks being provided to support residential and commercial development projects, the expansion of geodetic control networks based on SPCS27 in the more urbanized areas such as Louisville, Lexington, and northern Kentucky made utilization of SPCS27 more accessible to local surveyors in those areas.

It should be noted, however, that due to the lack of computational resources required to implement SPCS27, which during earlier times consisted mainly of trigonometric, logarithmic, and other supporting tables published by USC&GS, slide rules, and perhaps mechanical adding machines, coupled with the labor intensive and time consuming task of manually traversing long distances to control monuments tied to SPCS27, the implementation of SPCS27 for common tasks such as boundary retracement and topographic surveys for development projects in smaller cities and towns within the more rural areas of Kentucky was neither practical nor economically feasible. This condition persisted for several decades following the initial adoption of the SPCS until technological advances eventually made their way into the mainstream of surveying and mapping practice. The transition from the long standing legacy methods of conducting terrestrial surveys, which comprised the backbone of emerging aerial surveying and mapping technologies by establishing the required underlying ground control networks, to advanced automated methods brought about by the advent of the integrated circuit and digital processing ushered in a new era of surveying and mapping in which SPCS27 eventually played an indispensable supporting role.

## 1.2 - North American Datum of 1983 (NAD 83)

By the 1970s the Interstate Highway System, at least in Kentucky, was well under construction, if not substantially complete, and USGS had completed its topographic mapping of the Commonwealth at a scale of 1:24,000 for the 7.5 minute quadrangles. The vast national horizontal and vertical control network required to support both endeavors was now in place to support adoption of a modern global geodetic reference ellipsoid and associated horizontal datum with a target completion date of 1983. In 1986 the North American Datum of 1983 (NAD 83) was completed and based on the recently internationally adopted *Geodetic Reference System of 1980* (GRS 80) ellipsoid. Unlike the Clarke ellipsoid of 1866, which was geometrically derived through triangulation surveys, GRS 80 was defined as a surface achieving uniform gravimetric potential (Moritz, 2000) with NAD 83 being established on GRS 80 with geocentricity as a primary goal. The development of NAD 83 as a continental datum is comprehensively documented in the NGS publication *NOAA Professional Paper NOS 2 - The North American Datum of 1983* (1989).

From 1973 through 1976 a network of gravity base stations adjusted to the International Gravity Standardization Net 1971 was established to support extensive gravity surveys conducted across the Commonwealth (Keller, et.al., 1976 and Ammerman, et.al., 1978). This work resulted in the establishment of 33 gravity base stations, some of which anchored approximately 4,200 gravity measurements covering the eastern portion of the Commonwealth (generally east of 85 degrees west longitude). While it is not clear whether the results of the observations made by the gravity surveys performed during that period were incorporated into the overall model resulting in the GRS 80 definition, and hence a basis for NAD 83, the desire to incorporate gravity into the new datum definition along with contemporaneous work to acquire gravity observations at such a density is a clear indication that gravity had become a prime focus in the development of geodetic reference systems and datums.

As for SPCS on NAD 83 (SPCS83) as it applied to Kentucky, the Commonwealth was divided into two zones exactly as it had been for SPCS27, at least in a geographic sense (see Fig. 1.1.1). While there had been no attempt to establish the two zones in SPCS27 such that each zone occupied a unique region in coordinate space relative to the other (both zones were defined with false origins of  $x/\text{Easting} = 2,000,000 \text{ ft.}$  and  $y/\text{Northing} = 0 \text{ ft.}$ ), there was an attempt to separate the two zones within SPCS83 proper but not relative to SPCS27 in spite of false origins for SPCS83 being defined natively in meters. The result being that the Kentucky North and South Zones of SPCS27 and the Kentucky North Zone of SPCS83 all share a common spatial domain either in part or in whole, meaning that a coordinate pair representing a unique position within that spatial domain could not be differentiated between those three zones based solely on the coordinate values. Figure 1.2.2 below provides a coordinate space comparison between all NAD 83 SPCS zones for Kentucky.

Official adoption of SPCS83 in Kentucky was achieved on July 14, 1992 through the enactment of KRS 1:010 - *Legislative intent in establishing Kentucky Coordinate System of 1983* and the companion KRS 1:020 - *Kentucky Coordinate System of 1983* (KSPCS83). Through these two statutes the use of state plane coordinates, as realized on the Kentucky North and South Zones and defined on NAD 83, was declared a complete, legal, and satisfactory description for the

location of a survey station or land boundary corner. Although the parameters defining the false origins for KSPCS83 were defined in meters, the legally adopted meter to foot conversion factor specified for expressing coordinates was the U.S. Survey foot. Given it was general practice at that time to explicitly state the names and defining parameters for each zone exclusively on NAD 83 within the language of the statute itself, this narrowly defined approach resulted in the SPCS in Kentucky as it was previously defined on NAD 27 not being brought into the realm of a statutorily defined coordinate reference system.

While a stagnant technology regime hampered widespread adoption of SPCS27, at least during its first few decades, this was not the case for SPCS83. By the time NAD 83 had been established technological advancements on multiple fronts completely transformed the manner in which surveys were conducted on the ground as well as how the resulting maps and plats were produced in the office. Early advancements included the advent of commercially viable and affordable scientific handheld calculators, electronic distance measurement (EDM) devices, digitally enabled theodolites, and their subsequent integration into a single instrument. Although each of these advancements alone represented a leap forward in reducing the labor intensive process of traversing on the ground, particularly in rough and heavily vegetated terrain, the tightly *integrated total station* made it possible to traverse long distances in a reasonable amount of time using less resources with mathematical reduction of observations being accomplished on the fly.

In addition to advancements in how surveys were conducted in the field, equally transformational advancements were occurring in the office through the availability of affordable personal desktop computers equipped with survey coordinate geometry (COGO) and computer aided design and drafting (CADD) software as well as automated plotters and large format printers for the generation of digitally created maps and plats. Within ten years of SPCS83 having been adopted the concept of field-to-finish surveying had entered into the surveying vernacular. As personal desktop computers became cheaper to acquire and more powerful in their capabilities, as well as the ability to network them into enterprise systems, implementation of geographic information systems (GIS) by various state, county and municipal governmental agencies essentially made routine the requirement that geodetic networks be established and maintained on a continuing basis.

From the mid 1980's through the early 2000's the advent of the Global Positioning System (GPS) becoming available to the general public with the removal of *selective availability*, a technology that decreased the resulting accuracy of positions observed by non-military applications, allowed the use of SPCS83 to become embedded natively within the surveying framework, first through post-processing activities and later through on-demand real-time kinematic (RTK) capabilities. This development, along with continued improvements in digital processing and visualization brought about by increased computational capabilities, integration of COGO, CAD, and GIS technologies, as well as the development and dissemination of statewide digital vector data and raster imagery led to an inflection point in the usage of SPCS83 as an underlying framework for surveying, and in particular, mapping applications.

Advancements in digital data processing made it possible to digitize vast amounts of paper-based mapping data, much of it originally mapped on NAD 27, and convert to NAD 83 once in

digital form. Several data conversion projects in Kentucky resulted in statewide digital vector datasets being completed for political boundaries (state and county boundaries), roads (local, state, and federal), streams and lakes, soils, as well as digital raster imagery including Kentucky's first digital orthorectified aerial imagery (DOQQ) dataset and the digitally scanned USGS 7.5 minute quadrangles (Digital Raster Graphics or DRGs), both provided by USGS. This is in addition to the Topologically Integrated Geographic Encoding and Referencing (TIGER) data available in ESRI shapefile format through the United States Census Bureau since 1989. It was through these advancements that it became possible to establish a surveying or mapping project in SPCS83 (or SPCS27 for that matter) and integrate digital geographic data from several sources originally mapped using various reference frames into a common project file. While this worked well for small projects and areas of interest contained within one of the two Kentucky zones this soon became a problem for agencies whose interests covered the entire Commonwealth. This problem was further exacerbated by Kentucky also being divided into two zones (zones 16 and 17) within the Universal Traverse Mercator (UTM) system developed by the United States Army Corps of Engineers and utilized by USGS for publishing various digital geospatial products.

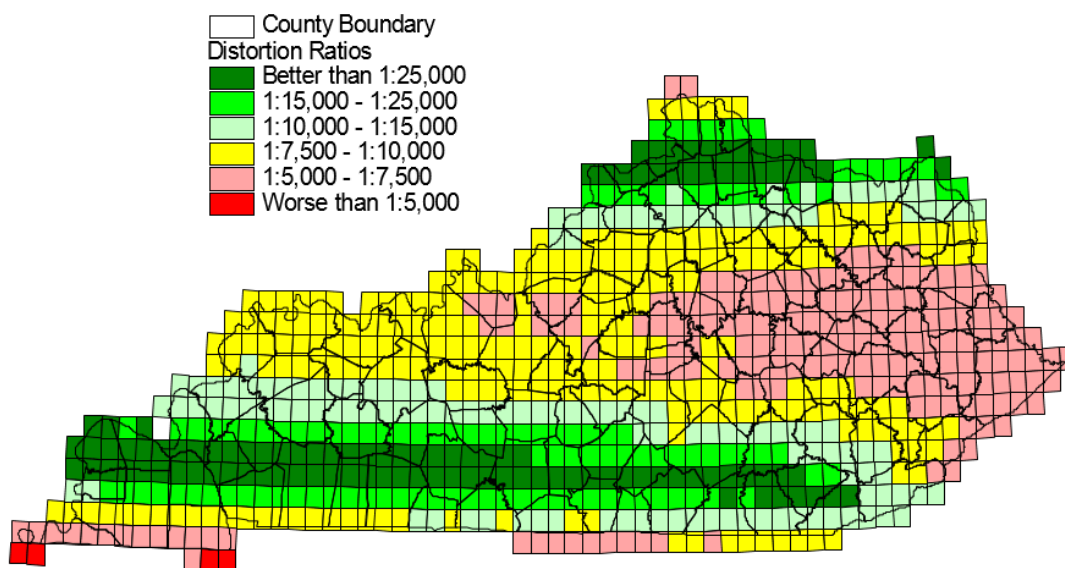
In August of 2000 the Kentucky Geographic Information Advisory Council (GIAC), a statutory advisory body established to inform and establish policy with respect to the use and implementation of geographic information throughout the Commonwealth, formed the One-Zone Subcommittee and charged it to review and interpret KRS 1:020 and the statute's impact on projections upon which the Kentucky Statewide Digital Basemap (KSDB) should be collected and/or disseminated and work with the various stakeholders in defining the issues around the need for a single zone projection for supporting the collection and dissemination of geospatial data comprising the KSDB. The subcommittee met from August through October of that year to develop a comprehensive solution to the multiple zone conundrum.

After considering several options, including extending the South Zone northward to cover the entire state (rejected due to the severe increase in mapping distortions as it extended northward), the subcommittee concluded that the best option was developing a new projection, to be called the Kentucky Single Zone, based on the following goals:

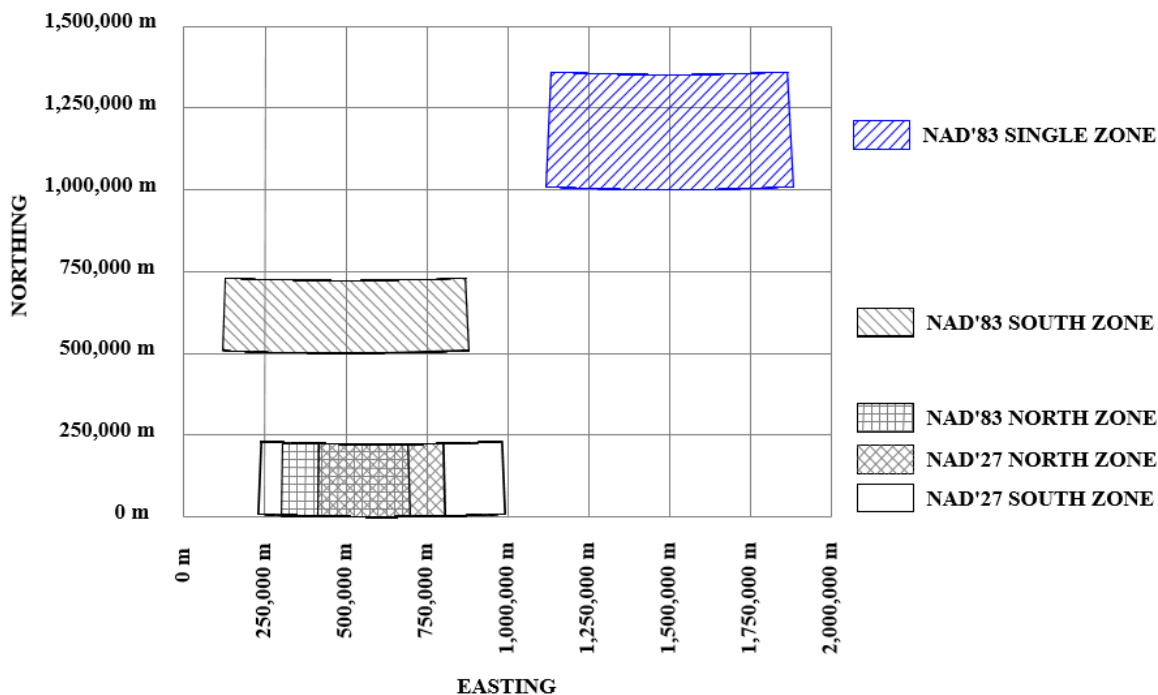
1. The Single Zone must be reasonably centered on the entire state to minimize distortions in a balanced manner and result in a projection suitable for land surveying and civil engineering endeavors located anywhere in the state, inasmuch as possible.
2. The goal of minimizing distortions would focus on those analyzed between the topographic surface and the projection grid. A desire to skew the projection northward to increase performance in the urban areas of northernmost Kentucky was expressed so long as all other parts of the state achieved a ground to grid distortion ratio of 1 part in 5,000 (200 ppm) or better (Figure 1.2.1).
3. The Single Zone projection would occupy a region in coordinate space such that any Single Zone position in Kentucky would result in a unique ( $x, y$  or N,E) coordinate pair not reproducible in any other zone, whether SPCS or UTM, for positions lying within or reasonably near the state boundary (Figure 1.2.2).

4. The process must not only include adoption by the GIAC, but also the promulgation of an administrative regulation establishing and adopting the Single Zone projection as an official means of representing spatial data and positions located within and reasonably near the Kentucky state boundary. Given there was an existing statute establishing the Kentucky Coordinate System of 1983, this promulgation will establish and adopt the Single Zone *in addition* to the existing North and South zones.
5. The Single Zone must be adopted and supported by the National Geodetic Survey and the United States Geological Survey through their line of products and services. It became clear that successful implementation and widespread acceptance of a new SPCS zone would require that Single Zone coordinates for National Spatial Reference System control monuments be published by the National Geodetic Survey.

On December 14, 2000 the full GIAC met and adopted the recommendations presented by the subcommittee (by that time internally referred to as the “Single Zone” subcommittee). During the following months the subcommittee developed language for an administrative regulation establishing and adopting the Kentucky Single Zone Coordinate System of 1983 (KY1Z), which was officially promulgated as 10 KAR 5:010 on August 15, 2001. By letter dated December 26, 2001 the National Geodetic Survey officially adopted the KY1Z as the primary zone for publishing its products and data to the general public for positions relevant to the Commonwealth of Kentucky.



**Figure 1.2.1** *Ground to grid distortion ratios for the NAD 83 Kentucky Single Zone (KY1Z) as originally computed during the design phase. The design process included evaluating the worst-case condition for each 7.5 minute quadrangle tile by applying the highest and lowest elevation occurring in each quadrangle to the centroid and computing their respective ground-to-grid distortions (Bunch, 2002).*



**Figure 1.2.2** Coordinate space comparison between the various Kentucky SPCS zones on NAD 27 and NAD 83, units in meters (Bunch, 2002).

Although not an original intent, once adopted the KY1Z represented the first instance of a layered system within the national SPCS due to Kentucky now having an original layer achieving statewide coverage through two separate zones and a subsequent layer, in a chronological sense, providing coverage as a statewide zone with both layers, in aggregate, covering the same geographic region. This arrangement would later be adopted by NGS as official policy for the State Plane Coordinate System of 2022 (SPCS2022) as discussed later in Section 1.7 below.

### 1.3 - Differences Between NAD 27 and NAD 83 (NADCON)

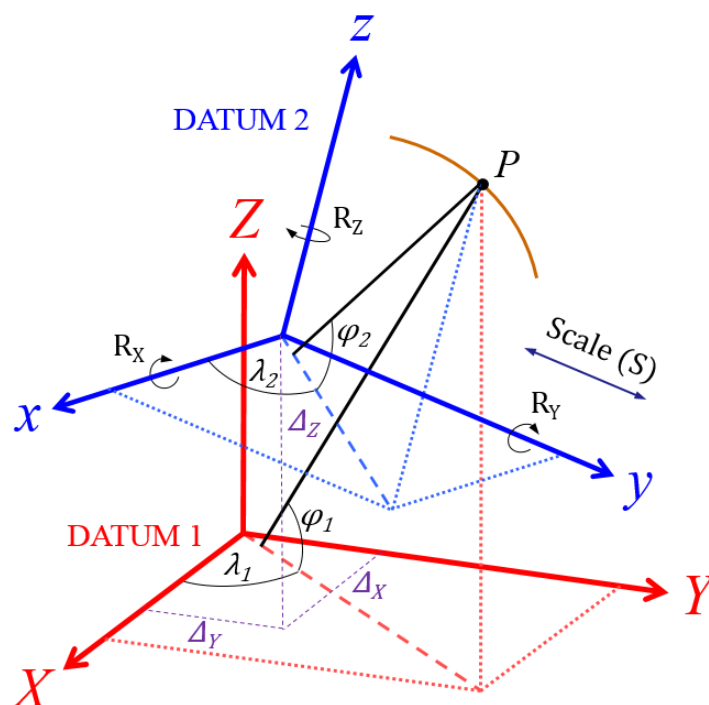
By their very nature geodetic datums are unique in some respect or another. Whether the differences lie between their respective size, shape, positioning, and/or orientation each datum is generally designed to serve a specific purpose or represents a refinement on a previous implementation. The end result is that they are invariably *different* and it is crucial that those differences be determined to a reasonable degree of certainty in order to maintain functionality between them, particularly when one supersedes another as NAD 83 does NAD 27. Thus, the purpose of a *datum transformation* is to define those differences and provide a mathematical process through which positions expressed in one datum can be confidently transferred to and from another datum as faithfully as possible to a reasonably known quantity of error.

Conceptually speaking, transforming coordinates between different datums should be a matter of discreet mathematical operation given the geometric nature in which modern datums are defined (generally an ellipsoid of rotation positioned by a fixed point and oriented relative to another fixed point or a series of azimuthal observations). Thus, in theory, one need only



determine how a source datum relates to a target datum by geometrically associating them through a 7 parameter Helmert transformation involving three translations ( $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$ ), three rotations ( $R_x$ ,  $R_y$ ,  $R_z$ ) and a scale factor ( $S$ ) (Figure 1.3.1).

The general problem lies not in how datums are defined, but in how they are *realized* on the ground. For instance, the Clarke 1866 ellipsoid was derived from triangulation surveys conducted primarily across the European-Asian continent (one arc was observed in Peru) and NAD 27 was *defined* by fixing a centrally located control station, Meades Ranch, Kansas Triangulation Station to a pre-defined position on the Clarke 1866 ellipsoid with orientation controlled by the various Laplace azimuths distributed through the network of arcs (Adams, 1930). Simple enough conceptually, however, NAD 27 was *realized* through the various interconnected triangulation surveys conducted across the continent and comprised of approximately 25,000 triangulation stations, several hundred base lines, and several hundred astronomical azimuth observations (NOAA NOS 2, 1989). This realization experienced local distortions resulting from network biases introduced by the complexities and limitations of physically performing and mathematically reducing triangulation surveys of that day. The result being that confidence in NAD 27 positions varied from region to region and was only as reliable as the underlying networks and computational regimens supporting them. All in all, however, NAD 27 was a product of the best available technology of the day and served its intended purposes well for decades following its implementation.

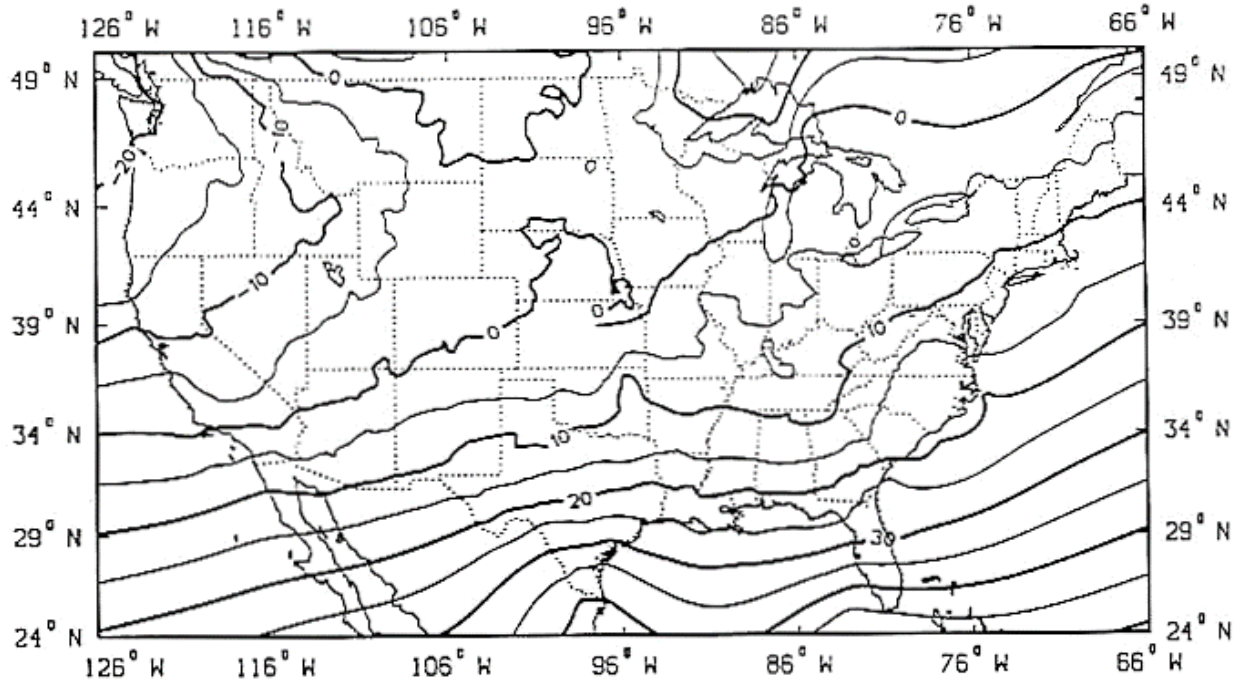


**Figure 1.3.1** Simple (and conceptual) geometric relationships between the Cartesian coordinate systems defining two datums, demonstrating how a single point ( $P$ ) is referenced relative to each datum and how one datum is related to the other through translation ( $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$ ), rotation ( $R_x$ ,  $R_y$ ,  $R_z$ ), and scale ( $S$ ).

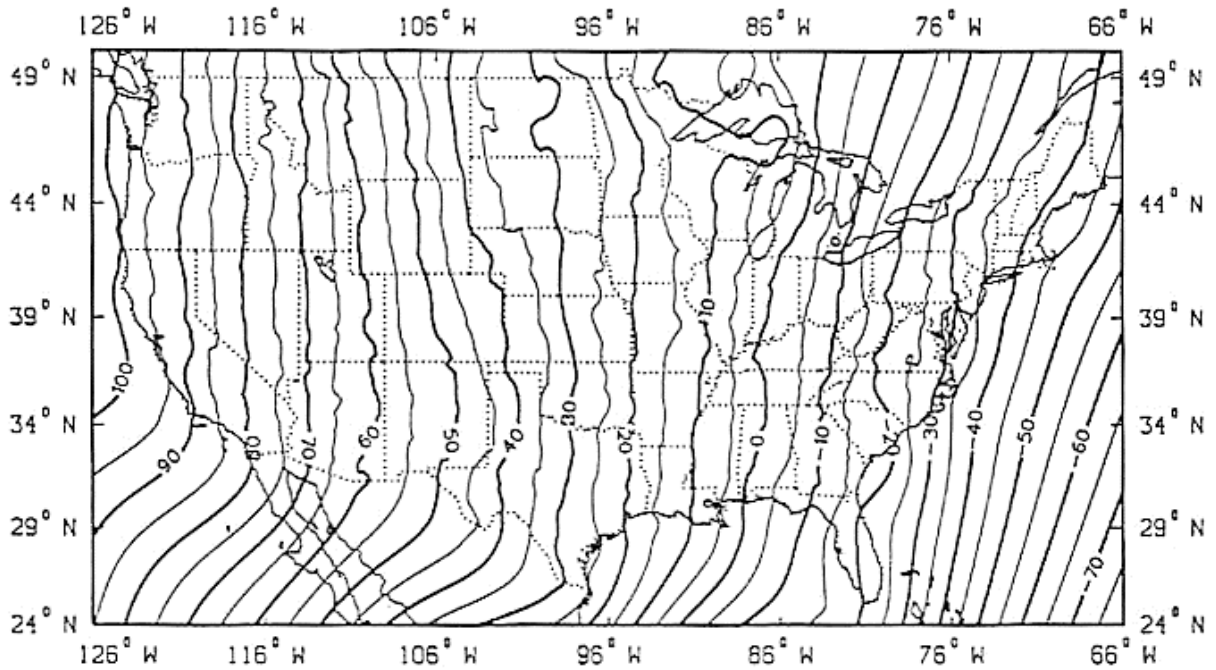
The takeaway here is that once datums are defined they have to be realized on the ground in order to be of practical use, otherwise they are just an exercise in academic conceptualization. While any given datum can have multiple realizations, generally based on refinements in the observational and/or computational framework upon which they are based, all realizations are still representative of their underlying datums (they don't represent new datums in and of themselves). This is an important distinction when working with datums and their realizations as they are refined and published over time.

By the time consideration for NAD 83 had become unavoidable the global observational network had been densified by an order of magnitude in comparison to that representing NAD 27. This was particularly the case for the continental United States (CONUS) thanks in large part to the establishment of geodetic control networks required to support mapping of USGS 7.5 minute quadrangle series and development of the Interstate Highway system. Thus, along with advances in geodetic observations such as Doppler point positions and very long baseline interferometry (VLBI), there were 1,785,772 geodetic observations connecting 266,436 control stations (Dewhurst, 1990), and approximately 30,000 EDM base lines and 5,000 astronomical azimuth observations (NOAA NOS 2, 1989) through which NAD 83 could be realized on the GRS 80 ellipsoid as determined by advancements in computational systems upon which refined statistical modelling techniques were applied, including simultaneous adjustment of the entire network (Dewhurst, 1990, p. 3). This combination established NAD 83 as a modern datum defined within an earth-centered, earth-fixed (ECEF) Cartesian coordinate reference system, meaning the GRS 80 ellipsoid was positioned to coincide geocentrically with the then computed center of mass of the earth and the orientation coincides with that of the Bureau International de l'Heure (BIH) Terrestrial System of 1984 (BTS-84) (NOAA NOS 2, pp. 82-84).

Given the stark differences between how the Clarke 1866 and GRS 80 ellipsoids were derived and how NAD 27 and NAD 83 were defined and realized upon those respective ellipsoids the task of developing a commensurate transformation regimen between the two datums proved to be a complex problem. Upon experimentation with various approaches NGS decided to use a grid-based method deemed better suited for capturing regional aberrations than the mathematically discreet Molodensky or Helmert transformation methods (Dewhurst, 1990, p. 7, Smith, et.al, 2017, p. 2). This approach resulted in the development of NADCON (for **N**orth **A**merican **D**atum **C**ONversion), a computer program originally released in 1990 by NGS for the purpose of transforming positions between NAD 27 and NAD 83, and predicated on two grids: one for expressing NAD 27 to NAD 83 shifts in latitude values and the other for expressing NAD 27 to NAD 83 shifts in longitude values. Each grid provided data points on one arc-minute intervals and prescribed a locally fit polynomial interpolation method, equivalent to bilinear interpolation, be implemented to compute predicted shifts at positions intermediate to those data points (Dewhurst, 1990, p. 16). Figures 1.3.2 and 1.3.3 demonstrate those shifts for CONUS respectively. Emphasis should be placed on understanding that these shifts do not represent movement of a point, particularly in a linear sense, as the point represents the same physical location on the ground regardless of how it is being referenced. These shifts represent variations in angular measurements as observed from *different origins of reference with each having their own directional orientation independent of the other*, as demonstrated in Figure 1.3.1 above.



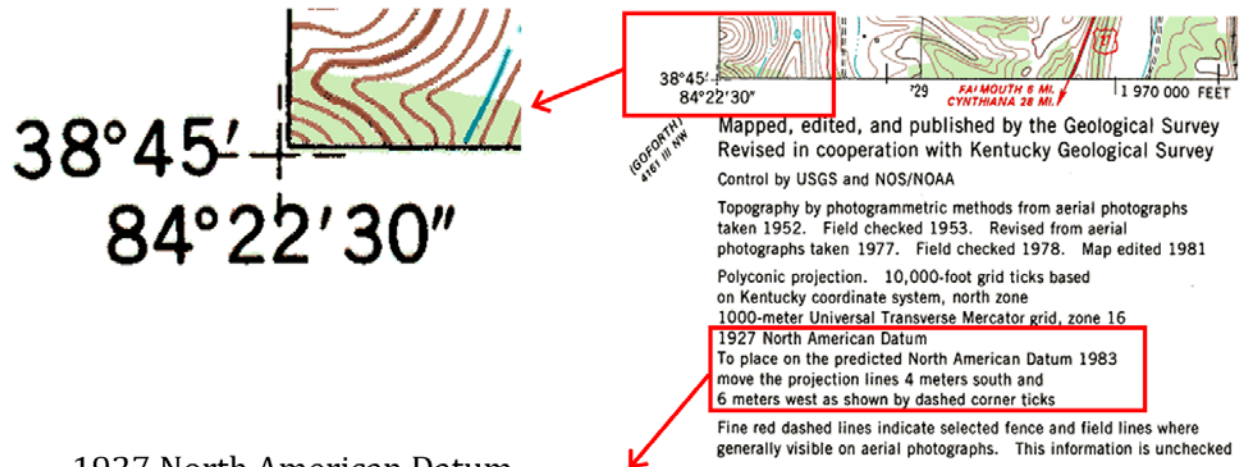
**Figure 1.3.2** Latitude shifts in CONUS in meters (NAD 83 minus NAD 27). (Dewhurst, 1990)



**Figure 1.3.3** Longitude shifts in CONUS in meters (NAD 83 minus NAD 27). (Dewhurst, 1990)

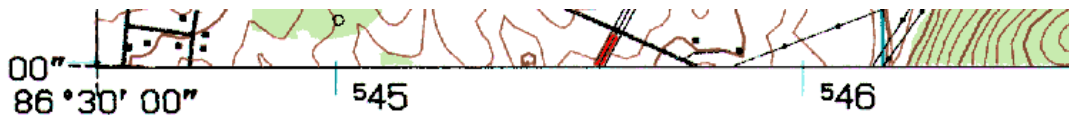
These shifts presented a practical problem for existing surveys and mapping products that were originally developed on NAD 27 but needed to remain relevant as NAD 83 became the standard. One such mapping product was the then ubiquitous 7.5 minute quadrangle series consisting of tens of thousands of map panels originally cast on NAD 27. This particular

conundrum was handled by USGS not by re-casting the mapping on NAD 83 but by providing datum shift indicators at the corners of the existing NAD 27 map collars (Figure 1.3.4). In addition to this “fix”, USGS eventually published later versions of a few quadrangles on NAD 83, some of which provided coverage in Kentucky. Figure 1.3.5 shows the lower left collar of the 1993 edition of the Bowling Green North, Kentucky quadrangle as it was published in 1996 and scanned in 2002 for the Kentucky Raster Graphic (KRG) dataset.



1927 North American Datum  
To place on the predicted North American Datum 1983  
move the projection lines 4 meters south and  
6 meters west as shown by dashed corner ticks.

**Figure 1.3.4** NAD 27 to NAD 83 datum shift as illustrated on the USGS 7.5 minute quadrangle series, originally cast on NAD 27. The dashed tick mark represents the NAD 83 position for the mapped NAD 27 quadrangle collar corner (solid border line).



**Produced by the United States Geological Survey**

**Compiled by photogrammetric methods from imagery dated 1953**

**Field checked 1954. Revised from imagery dated 1967**

**Field checked 1968. Revised in cooperation with State of Kentucky agencies from imagery dated 1993. Survey control current as of 1968. Map edited 1996. Contours not revised**

**North American Datum of 1983 (NAD 83). Projection and blue 1000-meter ticks: Universal Transverse Mercator, zone 16  
10 000-foot ticks: Kentucky Coordinate System of 1983 (south zone)**

**North American Datum of 1927 (NAD 27) is shown by dashed corner ticks. The values of the shift between NAD 83 and NAD 27 for 7.5-minute intersections are obtainable from National Geodetic Survey NADCON software**

**There may be private inholdings within the boundaries of the National or State reservations shown on this map**

*Figure 1.3.5 Lower left portion of collar for the 1993 edition of the Bowling Green North, Kentucky 7.5 minute quadrangle cast on NAD 83 and published in 1996. NAD 27 corner ticks are provided but in this version the shift distances are not given, referring to NADCON instead.*

#### 1.4 - NAD 83 Adjustments and Realizations, HARN, FBN, and Epochs (oh my)

When NAD 83 was published in 1989 it represented the completion of an epic task involving four countries (United States, Canada, Mexico, and Greenland) employing hundreds of people who over a period of 12 years evaluated survey observations dating back to the 1800's, vetted and digitized them into a centralized data model (which also had to be created), developed and/or refined computational mathematics and algorithms capable of simultaneously solving nearly one million equations resulting in the publication of 300,000 points, and relocated the adopted datum origin from a reference point on the surface of the earth to the earth's then known center of mass (Swartz, 1989; Vorhauer, Milbert, and Pursell, 2008, *p. 5*). Once the dust settled on this massive undertaking it was understood, even given the vast improvement the new datum represented over NAD 27, that based on the new and rapidly evolving satellite based Global Positioning System (GPS) the freshly minted reference frame would soon become obsolete unless additional adjustments were made in relatively short order. This state of affairs signaled the first usage of datum tags representing epochal designations for a given reference frame and the original version of NAD 83 was thus tagged *NAD 83(1986)*.

With GPS technology resulting in positional accuracies exceeding those achieved by terrestrial methods by several factors an incompatibility with NAD 83(1986) was realized as adjusted GPS surveys were being degraded to fit into the less accurate initial realization of the NAD 83 reference frame. To correct this situation NGS conducted subsequent surveys based on GPS technology to establish new control stations and refine published positions for existing control points. These initial GPS surveys comprised what would become the High Accuracy Reference Network (HARN) and were conducted on a state-by-state basis over several years from 1989 to 1997 which included likewise piecemeal readjustments of individual states resulting in

inconsistencies between them along their borders (Milbert, 1994), with Kentucky's portion being completed in 1993. In addition to the goal of refining the positional accuracy associated with existing control stations the HARN mission also established new control stations at denser intervals than previous surveys to improve availability, with station locations being established to support GPS surveying methods by minimizing obstructions to the horizon of orbiting satellites (Strange and Love, 1991). Once completed, it was routine to experience 0.5 meter shifts between the original NAD 83(1986) coordinates and the HARN readjusted positions.

The ongoing GPS technology juggernaut resulted in even higher positional accuracies with the introduction of Continuously Operating Reference Stations (CORS) in 1994. This refinement of the technology provided continuous positional observation over time with storage of the underlying data for later retrieval allowing post-processing analysis and adjustments for other GPS surveys conducted concurrently within the coverage area of one or more CORS. This rapid increase in positional accuracy created likewise problems for the HARN as had been experienced for the original realization of NAD 83(1986) relative to the early GPS surveys. By 1997 a new realization of NAD 83 had been completed and tagged NAD 83(CORS96) through which "positional distortion in the HARN relative to the CORS" had been identified resulting in the recommendation that a national readjustment of the HARN using CORS as the control be performed (Milbert, 1997).

With CORS expanded into a nationwide geodetic control network and tightly integrated into the NAD 83 (CORS96) realization, the basis for re-observing the HARN network in order to tie the two systems together and achieve tighter consistency between them had been established. From 1997 to 2004 a campaign to resurvey the HARN network based on a spacing of roughly 100 km using CORS as the control and achieving a relative horizontal accuracy of 1:1,000,000 was conducted. Once completed this effort, called the Federal and Cooperative Base Network (FBN/CBN), or Federal Base Network (FBN), represented the most accurate geodetic control network established for the NSRS, achieving horizontal accuracies at the 1 cm level, with an ellipsoid height accuracy of 2 cm (Vorhauer, Milbert, and Pursell, 2008, p. 8). In 2007 readjustment of the FBN resulted in the release of a new datum tag, called NAD 83(NSRS2007) and identified as NAD 83(2007) on the published NGS control datasheets.

From the early 1980s through the mid-to-late 1990s improvements in GPS accuracy were mainly brought about by commensurate improvements in the overall system itself, most notably through the addition of orbiting satellites until a full constellation of 24 satellites was completed. This rapid pace of improvement in the GPS infrastructure during the nascent years of NAD 83(1986) represented a hectic period for NGS with respects to maintaining the NSRS on the NAD 83 reference frame while keeping it relevant to the various activities it purported to support, hence the rapid development and deployments of HARN and CORS over the course of a single decade, with the FBN project being conceived and fully implemented within less than a decade afterwards. Two milestones reached during this period was the release of the NAD 83(CORS96) readjustment resulting from the establishment of a nationwide CORS network supported by a full complement of orbiting GPS satellites and the eventual reconciliation of the HARN network with CORS through the FBN program, thus culminating in the release of the NAD 83 (NSRS2007) realization with positions held fixed predominantly at the 2002.0 epoch.

Following the release of NAD 83(NSRS2007) improvements in positional accuracies achieved by the CORS network were mainly brought about through fine-grained refinements in receiver deployment and configuration, most notably related to "antenna calibrations, new/revised

processing algorithms, improved discontinuity identification, several years of additional GPS data, change in reference epoch, and an improved definition of the global reference frame, IGS08 ([geodesy.noaa.gov](http://geodesy.noaa.gov)). Based on these refinements subsequent updates to positions and velocities for all CORS resulted in the release of NAD 83(2011) in September of 2011 with positions held fixed to the 2010.0 epoch, which at the time of this writing (Spring, 2020) represents the latest refinement of the NSRS as realized on the NAD 83 datum.

## **1.5 – Kentucky Height Modernization and Continuously Operating Reference Stations (CORS)**

After adoption of the Kentucky Single Zone the need to address the vertical component of geodetic control started to come into focus within the Commonwealth. This attention was fostered by the continued evolution of GPS technology through which horizontal accuracies previously considered unachievable were becoming a routine matter of practice. While these new capabilities in 2-dimensional (horizontal) positioning were changing the landscape with regards to high-accuracy planimetric mapping and engineering projects, it remained an incomplete approach to providing geodetic control for projects requiring reliable vertical control. At that time the only way to acquire reasonably accurate elevation tied to the National Spatial Reference System for geodetic control was to run level lines across long distances mainly between legacy vertical control monuments installed during the 1930s through 1970s. As with the old “transit and chain” surveys for horizontal control this process was tedious, manually intensive, time consuming, and ultimately expensive. Given the level of accuracy obtained through GPS technology relative to an ECEF reference frame, it was considered possible to achieve relatively accurate ellipsoid height values, and in turn, derive elevations to within several centimeters when combined with the most recently published geoid model.

On September 19, 2002 the GIAC was presented with a proposal to establish the Height Modernization Subcommittee to explore the development of a reliable and accurate infrastructure for elevation data by modernizing and extending the existing vertical control network. This modernization would contribute to on-going development of refined hybrid geoid models through which higher levels of accuracy for elevation determination could be achieved. According to NGS “Hybrid geoid models are created by constraining a gravimetric geoid model to published heights using GPS observations on leveled bench marks.” Thus, in order to pursue height modernization the combination of old and new geodetic surveying techniques (high-accuracy leveling coupled with GPS observations) would have to be implemented. Given the continued importance of horizontal control and a desire to shorten the length of time required to obtain accurate GPS observations, the top priority for the height modernization effort was to establish a network of continuously operating reference stations (CORS) across the Commonwealth. This network would then be used to obtain accurate horizontal position and ellipsoid heights through GPS observations on existing and new bench marks installed during the Federal Base Network (FBN) program.

By 2006 the Kentucky Transportation Cabinet (KYTC) had deployed the Kentucky CORS network (KyCORS) consisting of 15 base stations, with all but one contributing to the NOAA CORS Network (NCN). These stations originally consisted of one in each of the 12 KYTC district offices and three state parks. By contributing to the NCN they also provided a framework

through which observations in Kentucky could acquire NGS Online Processing User Service (OPUS) solutions. In December of 2008 KYTC announced through *Design Memorandum No. 05-08* the availability of a service providing free single phase baseline corrections for horizontal positioning through anonymous access over the internet. By 2019 this network of base stations had grown to a total of 38 stations with 18 contributing to the NGS CORS network (Fig. 1.5.1).



**Figure 1.5.1** Kentucky CORS (KyCORS) network with stations in surrounding states. Green symbols represent stations within the NOAA CORS Network (NCN) and blue symbols represent stations within the KyCORS network only. (KYTC, 2019)

### 1.6 – Future Reference Frames Planned for 2022

At the time of this writing (Spring, 2020) the National Geodetic Survey is engaged in a multi-year project focused on modernizing the National Spatial Reference System by developing several terrestrial reference frames (TRFs) upon which the NSRS will be redefined. The planned geometric framework of this effort is documented in *NOAA Technical Report NOS NGS 62 – Blueprint for 2022, Part 1: Geometric Coordinates (NGS 2021 revised)* in which the primary focus is that of providing support for *time dependency*, predicated on the realization that geometric coordinates established for a control point will change over time due to the *direct and residual horizontal displacements* caused by plate tectonics and geological/geophysical conditions specific to a given region.



Since the scope of this document pertains to the SPCS in general this section will focus on functionality and characteristics targeted for the reference frames under development, which are:

- The *North American Terrestrial Reference Frame of 2022 (NATRF2022)*, which geographically covers the continental United States, including Kentucky and the focus of this document.
- The *Pacific Terrestrial Reference Frame of 2022 (PATRF2022)*.
- The *Caribbean Terrestrial Reference Frame of 2022 (CATRF2022)*.
- The *Mariana Terrestrial Reference Frame of 2022 (MATRF2022)*.

All of which will be defined on the GRS 80 ellipsoid.

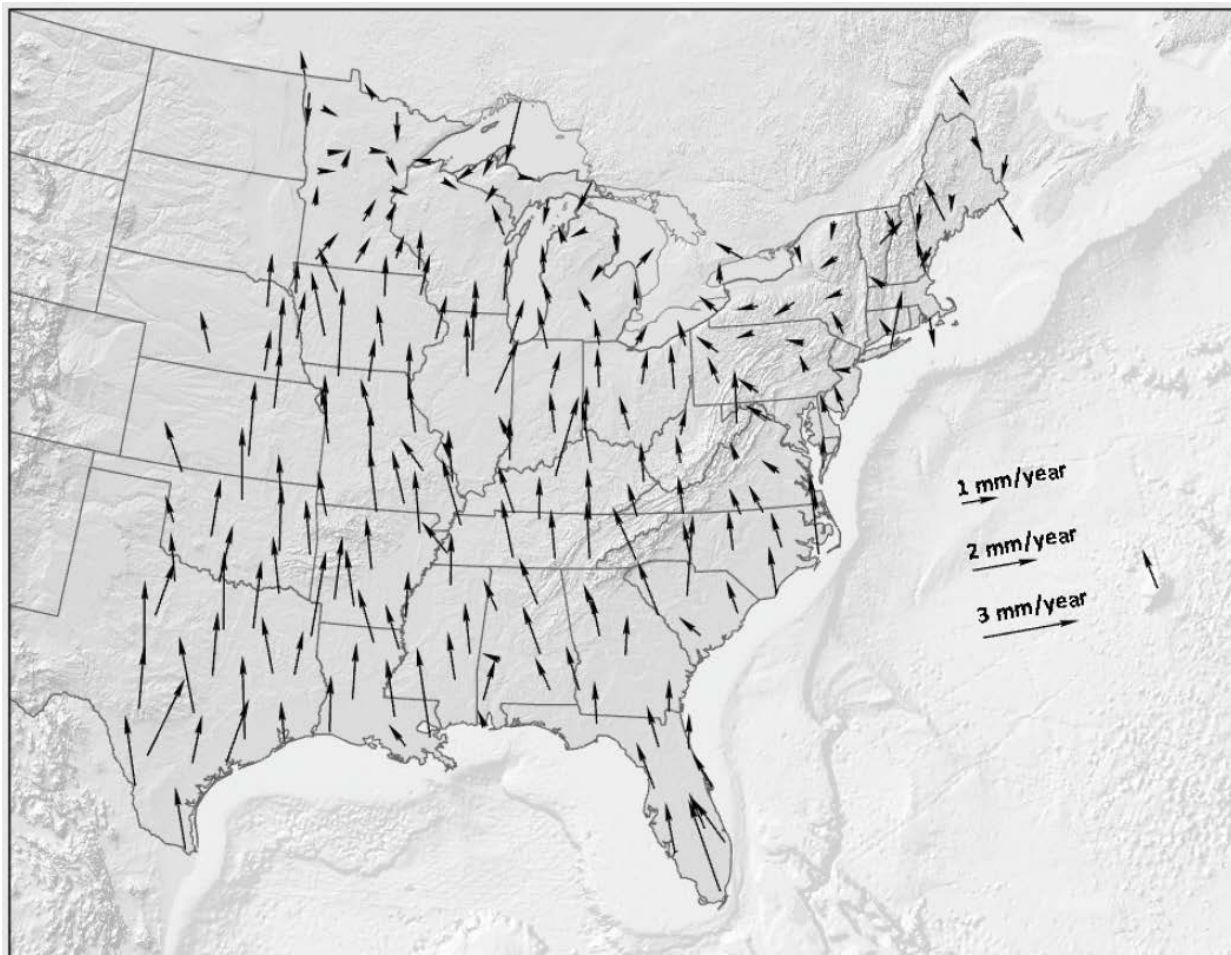
In the past, horizontal datums were realized through control networks in which positional accuracies achieved by the surveys defining them resulted in margins of error in excess of the cumulative movements of their underlying control points by several factors (even when considered over many years). Given the state of surveying technology today the NSRS, as planned for 2022, will need to support a framework in which routine observational methods can and do achieve accuracies well within residual motions occurring over just a few years, if not within a single year or two. In order to achieve this NGS has outlined a strategy that deals with time dependency associated with crustal motion by breaking down horizontal displacements into two components:

1. *“Plate-Fixed” motion* that conceptualizes overall motion as a rigid plate rotating about a geocentric axis, referred to as an “Euler [oi-leh] Pole” (Figure 1.6.1), passing through a fixed point on the surface of the earth, which is not necessarily located on the plate itself. The rotational motion is then expressed as a constant angular velocity ( $\omega$ ) about the pole resulting in increased linear velocities as distance from the pole increases. Once the location and angular velocity of the Euler Pole are established to a satisfactory degree of predictive accuracy, this component of motion can be accounted for through discreet mathematical transformation techniques where periodic refinement of  $\omega$  may be necessary over time, and by definition, involves no vertical displacement.
2. *Intra frame motion* that is determined by computing the residual motion left over once plate-fixed displacement as described above is removed from the difference between positions observed for the same point at time  $t = t_0$  and at some later time  $t = t_0 + \Delta t$ . Unlike the setup for plate-fixed motion, this component, termed *intra-frame velocity*, must be monitored and refined through observation over time and realized mathematically through advanced geostatistical analysis performed on a continuous or periodic basis. This regimen is being conceptualized by NGS as a service to be supported through an *Intra-Frame Velocity Model (IFVM)* based primarily on data collected through the Continuously Operating Reference Station (CORS) network.



**Figure 1.6.1** Vectors of horizontal velocity at select Continuously Operating Reference Stations (CORSs) on multiple tectonic plates determined by NGS, as well as an associated Euler pole solution for the North American Plate. (NGS 2021)

The above approach represents a compromise regarding the desire for observed coordinates to remain relatively constant within a given reference frame while acknowledging the fact that underlying control monuments, whether passive or active, will move over time. By introducing the plate-fixed component the strategy is to account for the vast majority of horizontal displacement within the overall framework by affixing the reference frame to the tectonic plate and allowing it to rotate in likewise fashion, thus reducing all plate motion associated with Euler Pole rotation to zero relative to the reference frame. This conceptually minimizes residual displacement of control points over time and relegates accounting for such displacement to the intra-frame velocity model component, at least where the assumption of rigidity is applicable to large areas of a tectonic plate and intra-plate interactions remain small or relatively predictable through monitoring and geostatistical analysis. Fortunately this is particularly true for Kentucky given its relatively stable location within the North American tectonic plate (with perhaps the notable exception of far westernmost portion lying along the New Madrid Seismic Zone). Figure 1.6.2 presents horizontal non-Eulerian velocities (movement after Euler-pole rotation has been removed) for the eastern portion of the North American tectonic plate, indicating a 1 to 3 mm shift per year for Kentucky.



**Figure 1.6.2** *Horizontal non-Eulerian velocities (observed minus Euler-derived) to the east of longitude 250°. Their magnitude is smaller than 2 mm/year. It is expected that those stations which were used to derive the Euler pole will behave well (have small non-Eulerian velocities) while other stations may have larger non-Eulerian velocities. (NGS 2021)*

As for planned deployment the initial configuration of the 2022 TRFs will be predicated on the following two conditions having been met (NGS 2021):

- Condition 1: The coordinate of any point in a plate-fixed frame should remain constant through time, if that point's only motion is a rotation about the Euler pole of that plate.*
- Condition 2: The coordinates of all points in a plate-fixed frame are identical to their coordinates in the global plate-independent frame at some initial chosen epoch  $t_0$ .*

In Condition 2 the concept of *ideal frame* refers to a globally established reference frame, such as the earth-centered, earth-fixed (ECEF) *International Terrestrial Reference Frame (ITRF)*, which is not tied to any specific tectonic plate thus making all observed positions time-dependent in nature. The result of these constraints will be that at some future epoch,  $t_0$ , geographic coordinates published on a 2022 TRF (the *plate-fixed frame*) will be the same as those published on the ITRF (the *ideal frame*) for the same control point, with differences

between them over time being accounted for through the applicable Euler Pole rotation model. It should be noted that non-Eulerian displacements will not be removed from published coordinates within the new reference frames, meaning subsequent intra-frame adjustments for residual displacements will be provided through a proposed IFVM service, which will presumably supersede or be integrated into the current *Horizontal Time-Dependent Positioning (HTDP)* service. The final take-away here is that coordinates provided and published *within a 2022 terrestrial reference frame* will remain stable over time while remaining *relatable* to the ITRF through discreet mathematical transformation, with adjustments for residual intra-frame displacements being accomplished through a subsequent service provided by NGS.

The above conceptualizations notwithstanding, there will be conditions on the ground where the underlying assumptions do not adequately account for all positional displacements over time, particularly along the edges of tectonic plates where rigidity no longer holds or in areas where regionalized geologic activity results in complex intra-frame movements, even across large areas. While these less than ideal conditions present a challenge to designing a framework in which positional displacements relative to a reference frame are minimized, preferably to a negligible degree over a reasonable period of time, it must be noted that overall functionality has to acknowledge practical limitations regarding scale and applicability simply as a practical matter. One noteworthy distinction when considering positional displacement is that of localized geologic phenomenon, such as soil creep and freeze-thaw action, which are clearly beyond the scope of any model or service to cover. It is therefore incumbent on the user to account for control monument placement and construction relative to long-term stability when considering the appropriateness of a positional time dependency regimen for any given project as supported by NGS through the NSRS.

Even with allowances for the above stated limitations the vast majority of spatially controlled projects, such as localized surveying and engineering endeavors, the geographic area of interest will be small enough such that relative displacements between control points will be negligible over time even without IFVM corrections and where positions are observed to a high degree of accuracy or displacements relative to the reference frame over time are somewhat tolerable. On the other end of the scale where the geographic area of interest may be large, such as supporting a state wide geographic dataset, the need to maintain high absolute positional accuracy over time is usually of secondary concern with intra-frame displacements being negligible *at the applicable scale* even when easily detectable. Either way, the introduction of an IFVM coupled with continuous or periodic monitoring and adjustment through an enhanced HTDP service should support even the most stringent cases where maintaining a high degree of absolute positional stability for originally observed positions *over time* is required to maintain spatial integrity for a desired outcome.

## 1.7 – SPCS Planned for 2022

### 1.7.1 – NGS Policy and Goals for SPCS2022

Modernization of the NSRS as planned for the 2022 release date includes reformulation of the State Plane Coordinate System for that framework (SPCS2022). While a great deal of work remains in the development of the 2022 terrestrial reference frames a significant amount of work has already been completed toward modernizing the SPCS for 2022, including the adoption of policies and procedures toward that end. The adopted changes and underlying reasons for them stand in stark contrast to the past transition from NAD 27 to NAD 83, particularly given the enormous technological changes and capabilities that have become routine since then, hence the need for modernization of this framework. This section represents a summarization of those policies and procedures, adopted and published by NGS as *State Plane Coordinate System of 2022 Policy (NGS Tracking Number 2019-1214-02)* and *Procedures for Design and Modification of the State Plane Coordinate System of 2022 (NGS Tracking Number 2019-1214-1-A2)*, both of which have been incorporated into this document as Appendix G and Appendix H respectively.

Changes made to SPCS policy for SPCS2022 include the aforementioned adoption of layered systems to be established for the individual states. This will be accomplished by establishing a default layer achieving statewide coverage through a single conformal mapping projection for each state and territory, to be designed by NGS with input from the respective states, and allowing the individual states to develop their own coverage schemes for up to two more layers: one being a multi-zoned arrangement achieving statewide coverage and another which achieves partial coverage of a given state to focus on urbanized and other regions of high value or special use within that state. Another tier of SPCS coverage includes an allowance, with caveats, for special zones that cover multi-territorial jurisdictions such as Native American tribal areas and reservations, federal jurisdictions, or greater metropolitan areas that span into two or more states.

In addition to allowing multi-layered arrangements SPCS2022 also focuses on optimizing performance between the projection plane and the topographic surface, or *ground-to-grid performance*. This represents a departure from previous policy in which design considerations mainly focused on mapping distortions relative to the defining ellipsoid. With this in mind, the stated policy for SPCS2022 includes redesigning the entire SPCS based, in part, on the following criteria:

- All states will be assigned a default layer comprised of a single zone designed to provide statewide coverage and optimized for ground-to-grid performance. This layer will be designed by NGS with input from the respective states when requested.
- All states will be allowed to propose and submit their own designs for secondary and tertiary layers that either achieve multi-zone statewide coverage or partial coverage through one or more zones, or both. However, only one multi-zoned layer can achieve statewide coverage exclusively, *or* only one layer can achieve partial coverage through one or more zones exclusively. The availability of these two layers represents an opportunity for the states to establish true low distortion projection (LDP) schemes for their respective jurisdictions.

- In cases where states have not submitted their own design schemes for SPCS2022 and are currently covered by multiple SPCS 27 and SPCS 83 zones NGS will design a default statewide multi-zoned layer predicated on their existing NAD 83 configurations but optimized for ground-to-grid performance.

In addition to the above generally stated criteria NGS policy includes the following technical characteristics and requirements, with emphasis added for significant departures from previous SPCS policies:

- All zones will be based on the Lambert Conformal Conic, Transverse Mercator, and Oblique Mercator projections with only a single mathematical form of each projection type as utilized by NGS being allowed.
- All map projections will be based on the GRS 80 ellipsoid as published without modifications and all input latitude and longitude values must relate to one of the four 2022 Terrestrial Reference Frames (TRFs), which must be identified for each zone.
- ***When specifying design criteria for given zones, the linear distortion criterion should be evaluated at the topographic surface, not the reference ellipsoid.***
- The unit of measure for defining all linear parameters shall be the meter, and all zones shall achieve positive coordinate values for all positions lying within them.
- All coordinate values will be published in meters. ***If coordinates in feet are provided, they will be labeled simply as “feet” and will be based exclusively on the definition of 1 foot = 0.3048 meter exactly (i.e., numerically identical to the International Foot).***
- Latitudes are positive in the northern hemisphere and negative in the southern hemisphere. ***Longitudes are positive east from the prime meridian (0° to 360°).***

The above being a general summary, the reader is directed to the linked policy and procedures documents presented in Chapter 3 (p.53) of this document below for a more complete narrative on this topic.

### 1.7.2 – SPCS2022 in Kentucky

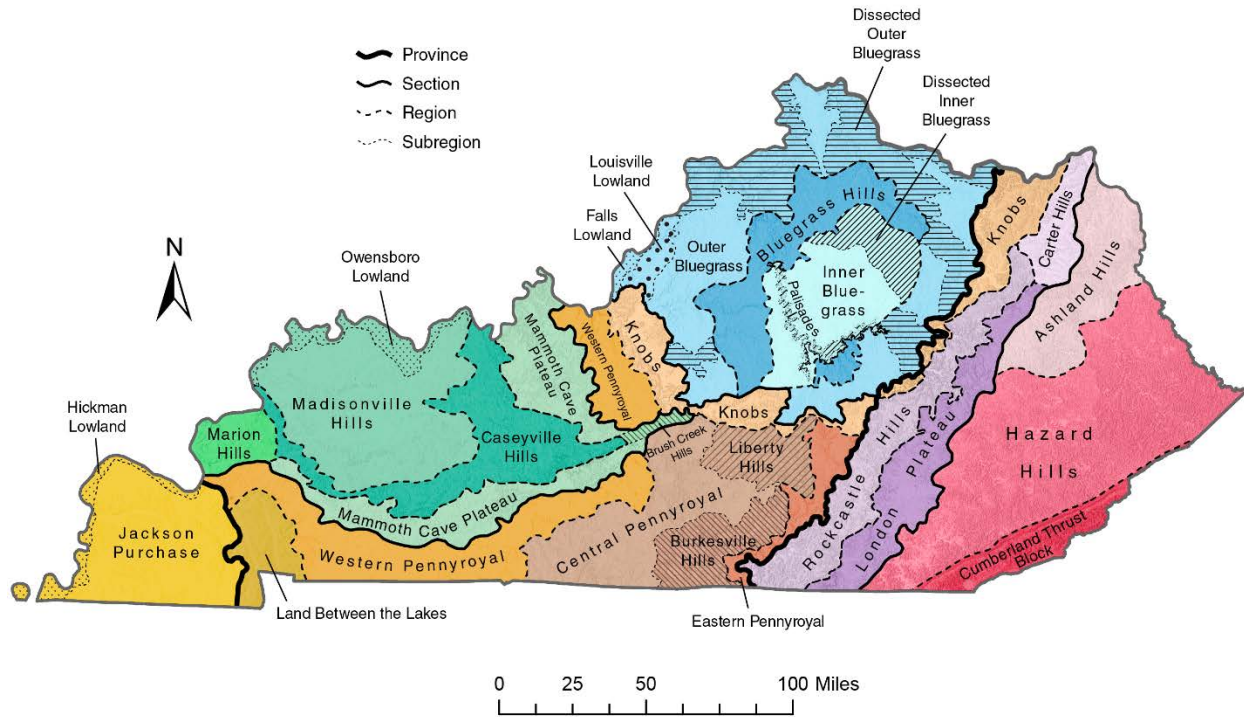
In July of 2019 the Kentucky GIAC established the Kentucky State Plane Coordinate System Subcommittee and charged it with reformulating the entire Kentucky SPCS in anticipation of commensurate changes being made at the national level by NGS. The subcommittee met during the months of September and October of that year resulting in the following goals and criteria:

- The entire SPCS in Kentucky would be redesigned to achieve ground-to-grid optimization for all layers established during this effort.
- To achieve further optimization the original multi-zone coverage comprised of the North and South zones would be deprecated and replaced with a multi-zone low distortion projection (LDP) scheme. In addition to the LDP layer, the KY1Z statewide projection will be redesigned to further optimize ground-to-grid performance based on the recently completed high-resolution digital elevation model (DEM) acquired through LiDAR technology.
- Each new zone established for the LDP layer will be defined along statutory or administrative boundaries comprised of multiple whole counties such that each county

lies exclusively within a single zone. An attempt will be made to group counties within the same physiographic region or those sharing similar characteristics with an overall goal to minimize the total number of zones while optimizing projection performance for each zone.

- Map projection parameters expressed in angular units of measure would be established to the nearest three 3 whole arc minutes to avoid repeating decimal values. Skew azimuths for the oblique Mercator projection method would be established to the nearest 5 whole degrees (although final design configurations ended up being established to the nearest whole 10 degrees).
- The false Northing and false Easting values established for a zone origin would be expressed to the nearest whole 5,000 meter increment.
- Each new zone, regardless of parent layer, would occupy a unique region in generic coordinate space separated at a minimum by 10,000 meters. This separation includes all previous zones for all previous series (datums) and zones defined within the Universal Transverse Mercator (UTM) system.
- The legacy U.S. Survey Foot definition applicable to the KSPCS would be deprecated and replaced by the International Foot definition for the 2022 series and all future series thereafter.
- The Kentucky statute defining the KSPCS (KRS 1:010 and KRS 1:020) will be revised and reformulated to define the KSPCS in a more generic sense while delegating definition of the finer-grained details to the administrative regulatory framework (KAR). The concept of defining the KSPCS as a *series of layered zones* would be encapsulated directly into the language, which would allow the SPCS as defined on all datums to be adopted within the statutory framework.
- The Kentucky Administrative Regulation (KAR) originally defining the Kentucky Single Zone Coordinate System of 1983 (10 KAR 5.010) would be completely revised and re-written to cover the entire KSPCS as it is defined across all datums. In order to facilitate timely revisions to the KSPCS commensurate with changes made at the national level or administrative needs arising at the state level the *KSPCS Standards and Specifications Document* (this document) would be incorporated by reference into the regulatory framework. *Addendum: Due to executive branch reorganizations over the years KSPC2022 will be implemented through promulgation of 200 KAR 041:010. Thus, 10 KAR 5:010 will expire and become obsolete.*

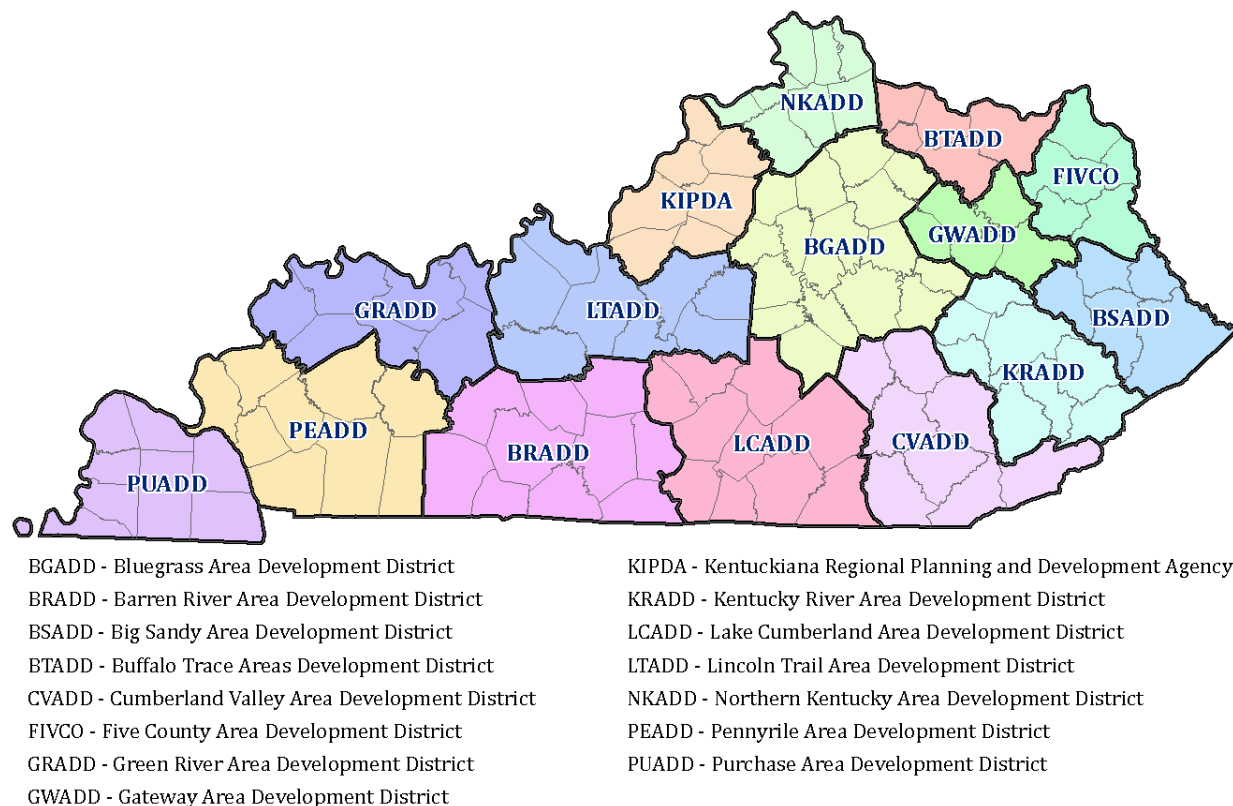
The task of designing the KSPCS2022 layers occurred concurrently with subcommittee deliberations and resulted in a refined single statewide zone (KY1Z2022) for the first layer and seven low distortion zones achieving statewide coverage for the second layer, thus replacing the legacy North and South zone arrangement. The design process included an analysis of the Kentucky physiographic regions relative to various administrative district boundaries in order to optimize projection performance relative to the topographic surface. The Kentucky Transportation Cabinet's (KyTC) highway districts were given first consideration based on that agency's reliance on the KSPCS for design and construction of Kentucky's highway system and heavy investment in the Commonwealth's transportation infrastructure, however, the aggregate size and distribution of counties therein resulted in configurations too large or unwieldy to achieve an optimal LDP outcome.



**Figure 1.7.2.1** Landform regions map of Kentucky. (Andrews, Morris, 2020)

Having failed to justify using the KyTC highway district boundaries as the basis for the LDP layer the focus quickly turned to identifying a scheme that more closely aligned with Kentucky’s physiographic regions based on the premise that optimizing ground to grid performance would be facilitated by considering regional topographic characteristics (Figure 1.7.2.1). Based on this criteria there was one district grouping that stood out above the rest in this regard: Kentucky’s Area Development Districts (ADDs), shown in Figure 1.7.2.2, which is not surprising given the correlation between physical geography and economic development, a principal goal in creating the ADDs. Another contributing factor for this regionalization scheme is that the Commonwealth is divided into 15 distinct ADDs, rendering their size and shape to be more suitable for mapping projection coverage than other administrative geographies comprised of fewer districts. Also, the correlation between ADD boundaries and physiographic regions coupled with their respective sizes and shapes is significant enough that the final design scheme resulted in all but one ADD being merged with at least one other adjacent district to comprise a given LDP zone (Figure 1.7.2.3).

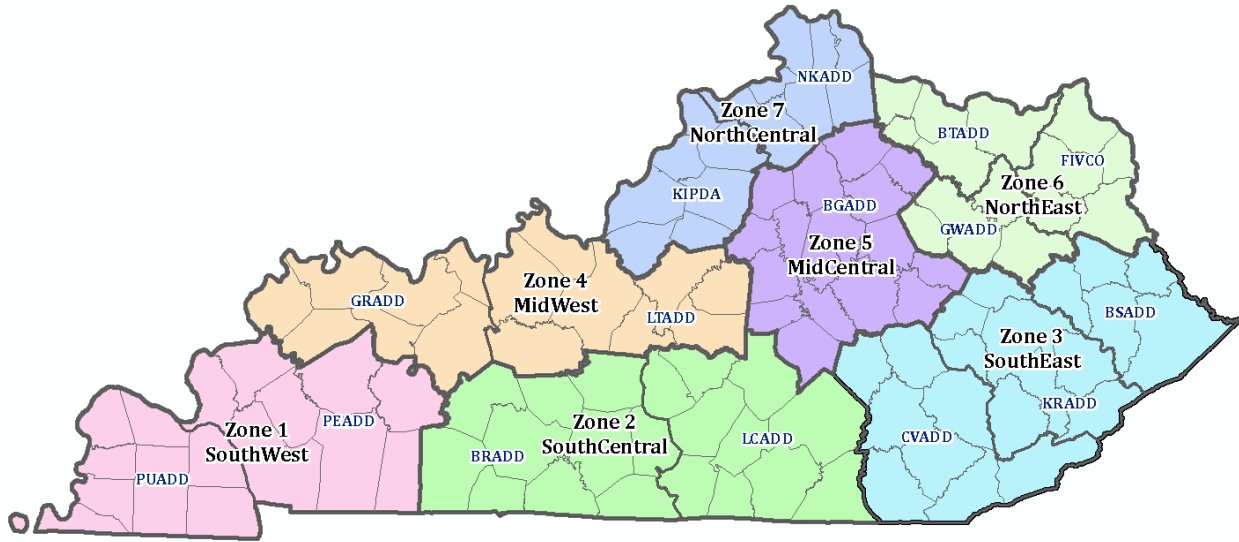




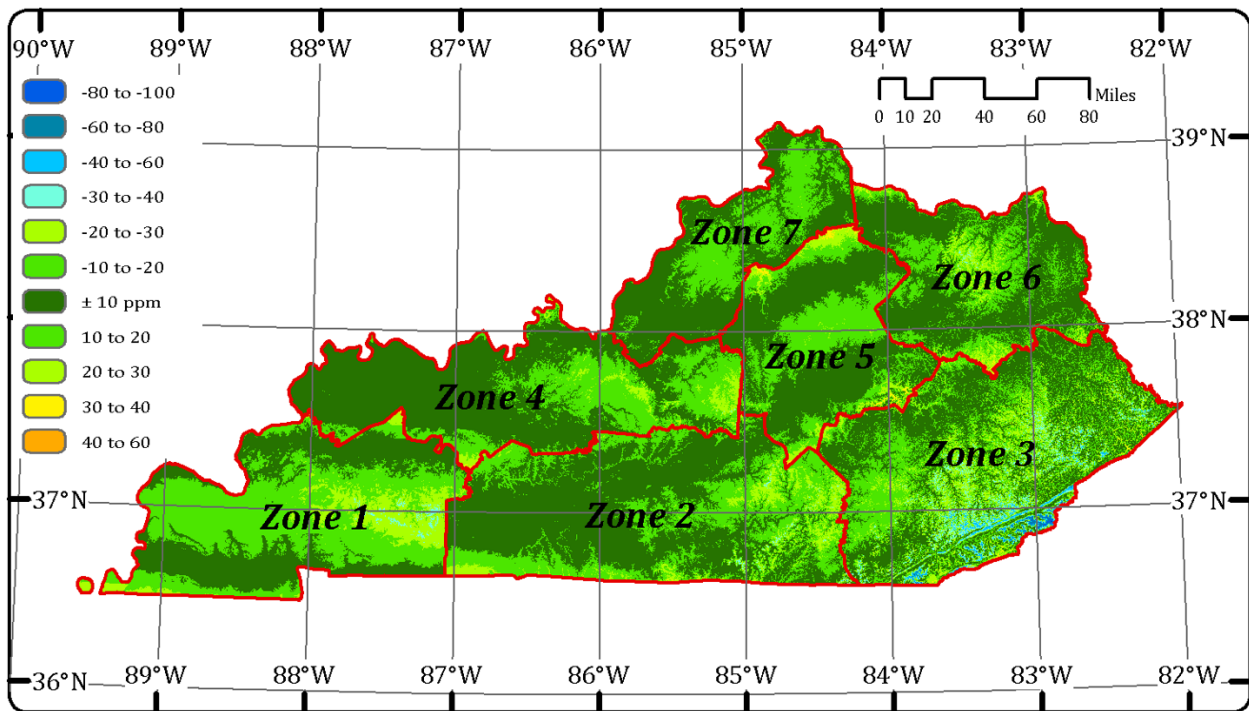
**Figure 1.7.2.2** Kentucky's 15 Area Development Districts (ADDs).

The task of quantifying a successful LDP regimen started with prioritizing overall design criteria by holding coarse grained criteria fixed while being somewhat flexible on the finer level constraints. For instance, it was determined that adhering to administrative grouping of county boundaries and minimizing the overall number of zones would take higher precedence over the percentage of a zone that achieves a given distortion outcome, albeit flexibility would be constrained to within five percent of a quantified criteria. Based on that approach the general performance criteria with respect to distortion required that at least 50 percent of a zone boundary achieve 20 parts per million (ppm) or less while keeping 80 percent of the zone boundary to 30 ppm or less. In addition to the overall zone boundary performance a goal of achieving 20 ppm or less for at least 85 percent of all grid cells lying within census blocks comprising the densest 90 percent of a county's population was also included in the performance analysis, with grid cells being established on a six arc-second spacing. Each grid cell was assigned the ellipsoid height, in meters, based on the latest statewide digital elevation model (DEM) as determined by light-detection and ranging (LiDAR) technology and computed using the latest geoid model published by NGS (GEOID 12b), with the overall vertical accuracy with respect to the ellipsoid height estimated to be between 0.3 and 0.5 meter.

Final design configuration for the LDP layer resulted in seven total zones based on combining ADD boundaries where possible with each zone achieving the above stated criteria within the flexibility tolerances (Figure 1.7.3). Four zones were defined using the Lambert Conformal Conic (LCC) projection type while three zone were defined using the oblique Mercator (OM) projection type. Final projection performance is shown in Figure 1.7.2.4 in aggregate as a statewide layer. Table 1.7.2.1 provides a quantified summary of overall performance for each zone.



**Figure 1.7.2.3** Final zone configuration for the KSPCS 2022 LDP layer based on combining Area Development District (ADD) boundaries.



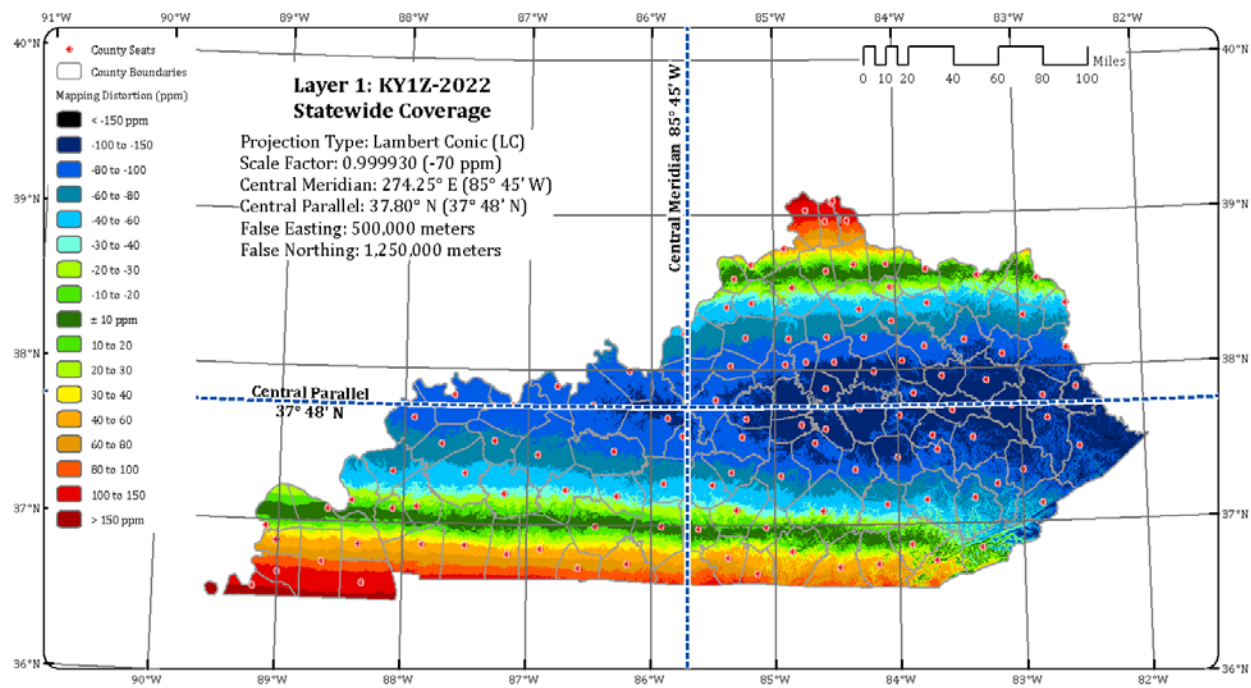
**Figure 1.7.2.4** Projection performance achieved by the seven LDP zones in aggregate statewide.

Modifying the original Kentucky Single Zone projection to achieve optimized performance relative to the topographic surface was accomplished by removing the original constraint of favoring northernmost Kentucky established for the original single zone projection (KY1Z 1983) and balancing overall distortions to be equally distributed geographically and in

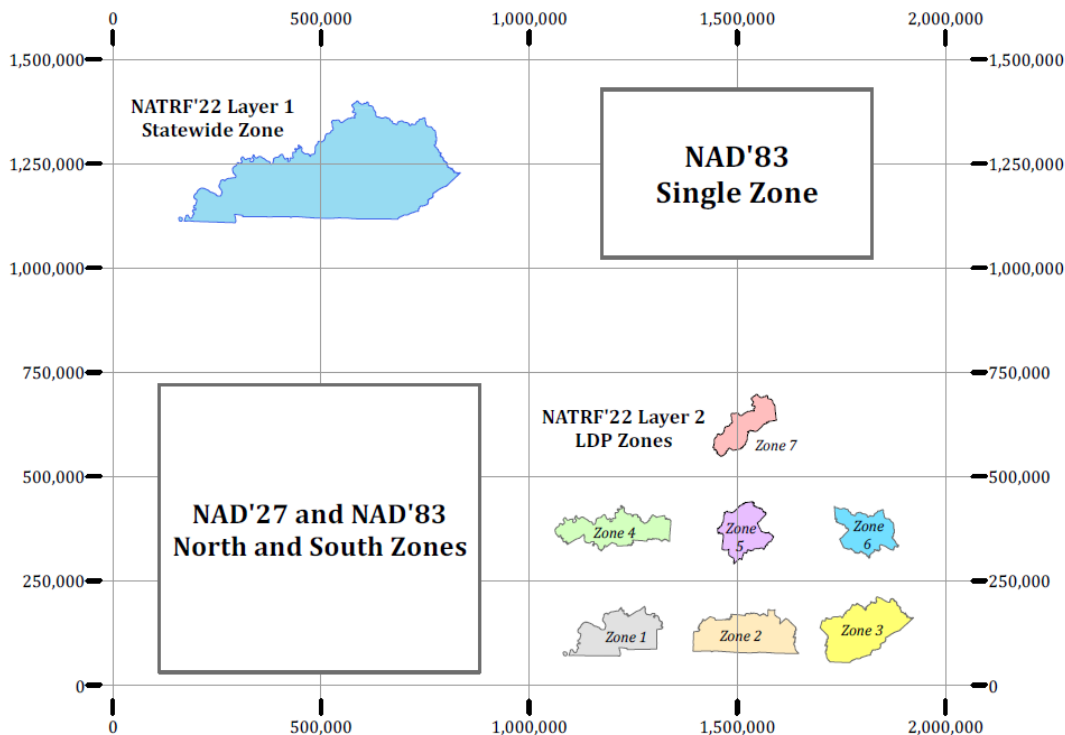
magnitude inasmuch as possible. The final configuration for the single zone layer resulted in improved performance in the central easternmost portion of the state where the projection plane achieves maximum distance below the topographic surface and performance improvements were likewise experienced in the southwestern part of the state where the projection plane achieves maximum distance above the topographic surface. Performance results for the 2022 statewide single zone layer are shown in Figure 1.7.2.5.

**Table 1.7.2.1** Map projection distortion performance for KSPCS 2022 layers and zones.

Layer	Zone Number	Zone Name	Projection Type	Percent Zone Area at 20 ppm or less	Percent Zone Area at 30 ppm or less	Percent of 0.9 Population by Density at 20 ppm or less	Minimum Distortion (ppm)	Maximum Distortion (ppm)	Mean Distortion (ppm)	Median Distortion (ppm)	Std.Dev. Distortion (ppm)
Statewide Single	1	Single Zone	LCC	-	-	-	-167	191	-40	12	65
Statewide Multi (LDP)	1	SouthWest	LCC	75.6	89.9	88.5	-37	66	-1	14	20
Statewide Multi (LDP)	2	SouthCentral	LCC	72.3	85.1	95.4	-52	74	7	11	20
Statewide Multi (LDP)	3	SouthEast	OM	67.1	81.0	84.9	-102	82	3	-10	24
Statewide Multi (LDP)	4	MidWest	LCC	70.6	83.4	97.6	-45	65	9	10	19
Statewide Multi (LDP)	5	MidCentral	OM	66.1	75.5	93.3	-32	88	13	28	23
Statewide Multi (LDP)	6	NorthEast	LCC	66.4	76.4	92.8	-37	93	12	28	25
Statewide Multi (LDP)	7	NorthCentral	OM	93.1	99.5	99.9	-24	41	1	8	11



**Figure 1.7.2.5** Distortion performance achieved by the KSPCS 2022 statewide single zone layer.



**Figure 1.7.2.6** Coordinate space comparison for all KSPCS 2022 zones, with those defined on pre-2022 datums represented by their respective extents and in aggregate where they overlap.

Upon completion of the design phase language for revising the relevant statute and Kentucky administrative regulation was formulated, with the full GIAC approving the KSPCS subcommittee’s recommendations to proceed with submitting the final designs to NGS for approval and revising KRS and KAR accordingly. On October 28, 2019 Kentucky submitted the relevant proposal forms to NGS and received approval to proceed with design submittal on November 1<sup>st</sup>, with subsequent approval and acceptance of the designs being acknowledged on November 8<sup>th</sup>. Based on refinements to the analytical processes developed for the design phase of this project a modified design configuration for Zone 5, involving revising the projection type from the transverse Mercator method to the oblique Mercator method, was resubmitted on December 3<sup>rd</sup> which was subsequently approved on December 26, 2019.

Following this final approval the language comprising the KRS was submitted to the Kentucky Legislature and introduced as House Bill 302 for the 2020 General Assembly legislative session on January 21, 2020. The bill was passed unanimously by the House on February 20<sup>th</sup> and received in the Senate the next day. On March 26<sup>th</sup> the bill passed unanimously in the Senate and was subsequently signed by the Governor on April 7<sup>th</sup>, to take effect on July 15, 2020.

## Chapter 2: Mapping Projections and SPCS Design Techniques

### 2.1 - Map Projections

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#### 2.1.1 - Map projection types and conformality

When a map projection is associated with a specific geodetic datum (i.e., geometric reference frame or geographic coordinate system), it is called a *projected coordinate systems* (PCS). A PCS definition must always include a projection type, geodetic datum, and linear unit.

Thousands of map projection types have been developed, and about a hundred are commonly used for a wide range of geospatial applications. Fortunately, the list of projections appropriate for surveying and engineering is much shorter, because they should be:

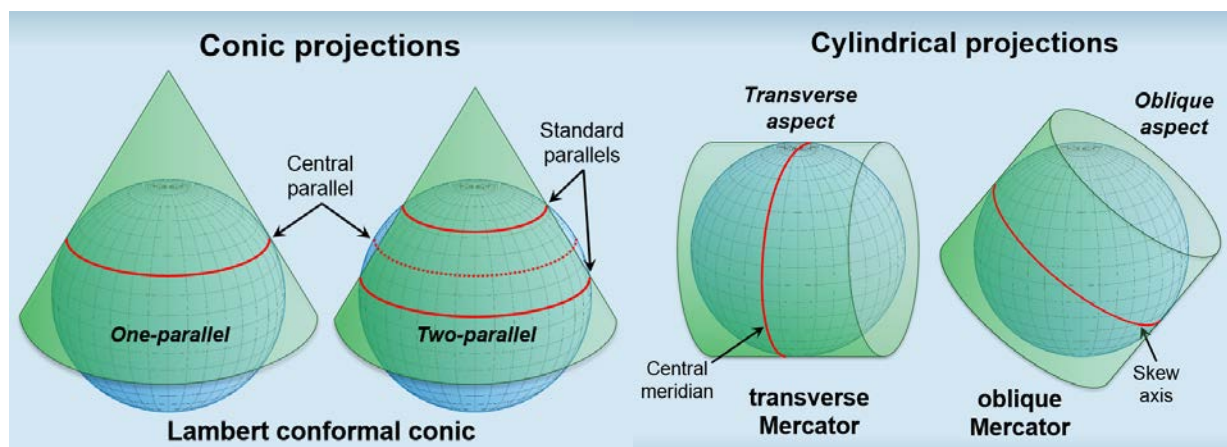
1. Appropriate for large-scale mapping (i.e., not just for covering large portions of the Earth)
2. Widely available and well-defined in commercial geospatial software packages
3. Conformal

Based on these three criteria, the number of conformal map projections adopted for usage in the State Plane Coordinate System reduces to the three listed in Table 2.1.1: transverse Mercator (TM), Lambert conformal conic (LCC), and oblique Mercator (OM). These projections types are shown in Figure 2.1.1 (note that throughout this document, OM always refers to the Hotine version of this projection).

**Table 2.1.1** Conformal projections used for the State Plane Coordinate System

Projection	Type	Usage*	Comments
Transverse Mercator (TM)	Cylindrical	SPCS, UTM	Often used for areas elongate in north-south direction. Perhaps the most widely used projection for large-scale mapping. Also called the Gauss-Krüger projection.
Lambert Conformal Conic (LCC)	Conical	SPCS	Often used for areas elongate in east-west direction. Also widely used for both large- and small-scale mapping. Includes both the one-parallel and two-parallel versions (which are mathematically identical).
Oblique Mercator (OM)	Cylindrical	SPCS	Often used for areas elongate in oblique direction. Not used as often as the TM and LCC projections, but widely available in commercial software. A common implementation is the Hotine OM (also called “Rectified Skew Orthomorphic”).

\*SPCS = State Plane Coordinate System; UTM = Universal Transverse Mercator



**Figure 2.1.1** SPCS map projection developable surfaces and their projection axis.

For all non-conformal projections (such as equal area projections), meridians and parallels generally do not intersect at right angles, and scale error varies with direction, so there is no unique linear distortion at a point. These characteristics make non-conformal projections inappropriate for most surveying and engineering applications.

The “flat” surface upon which coordinates are projected is called the *developable surface*. There are two types associated with the SPCS, cylinder and cone, as shown in Figure 2.1.1. Each of these is “flat” in the sense that it can be represented as a plane without distortion, because it has an infinite radius of curvature in at least one direction. Conceptually, the cylinder and cone can be “cut” parallel to their central axis (which is the direction of infinite radius of curvature) and laid flat without changing the relationship between the projected coordinates. Another way to think of it is that there is only one developable surface, the cone: a cone of infinite height is a cylinder, and a cone of zero height is a plane.

Each of the projection types listed in Table 2.1.1 has a specific set of five to seven defining parameters. One is  $k_0$ , the projection scale (factor) on the *projection axis*. The projection axis is the line along which projection scale error is minimum and constant with respect to the reference ellipsoid, as shown in Figure 2.1.1. It is the *central meridian* ( $\lambda_c$ ) for the TM, the *central parallel* ( $\varphi_c$ ) for the LCC, and the *skew axis* for the OM. Actually the scale is not quite constant along the OM skew axis but exactly equals  $k_0$  at a single point (the local origin) and changes slowly along the axis with distance from the origin (it usually differs from  $k_0$  by less than 1 ppm within 500 km of the local origin). For the two-parallel LCC,  $k_0$  is defined as less than 1 implicitly, by the distance between the north and south standard parallels (the further apart the standard parallels, the smaller is  $k_0$ )

The  $k_0$  value defines the scale relationship between the ellipsoid and conformal developable surfaces, as listed below and shown in Figure 2.1.2:

- **$k_0 < 1$ .** The developable surface is inside (“below”) the ellipsoid and intersects the ellipsoid along two curves on either side of the projection axis, beyond which the developable surface is outside (“above”) the ellipsoid. In this case the projection is called *secant* because it cuts through the ellipsoid. Secant is the most common projection configuration for published PCSs (such as SPCS and UTM) because it covers the largest region with the least absolute scale error with respect to the ellipsoid. Positive and negative scale errors are balanced for secant projection zone as shown in Fig. 2.1.2, with approximately the middle 71% of the developable surface below the ellipsoid and the outer 14.5% on either side above the ellipsoid. The “zone” is the area on the Earth where the PCS is used.
- **$k_0 = 1$ .** The developable surface is tangent to the ellipsoid. That is, it touches the ellipsoid along its projection axis.
- **$k_0 > 1$ .** The developable surface is above the ellipsoid and does not intersect the ellipsoid surface anywhere. This approach is often used to place the developable surface near the topographic surface, which is typically above the ellipsoid. The intent is to decrease linear distortion of the projected coordinates with respect to the ground surface, rather than the ellipsoid surface.

In addition to the projection axis scale, at least four other parameters are needed to fully define the projections listed in Table 2.1.1. Two of these are the latitude and longitude of its *geodetic origin* ( $\varphi_0, \lambda_0$ ). The geodetic origin may or may not be located on the projection axis. It is always on the central meridian of the TM ( $\lambda_0 = \lambda_c$ ) but often is not on the central parallel of the LCC projection ( $\varphi_0 \neq \varphi_c$ ), in which case at least six parameters are required to define an LCC. The other two parameters are the projected coordinate values of the geodetic origin, often called the *grid origin* and specified as *false northing* ( $N_0$ ) and *false easting* ( $E_0$ ) in this document. Grid origin values are usually selected such that projected coordinates are positive within the zone. An additional (sixth) parameter called the *skew axis azimuth* ( $\alpha_0$ ) is required for the OM projection to specify the orientation of its skew axis ( $\alpha_0$  can also be defined implicitly by using a two-point definition).

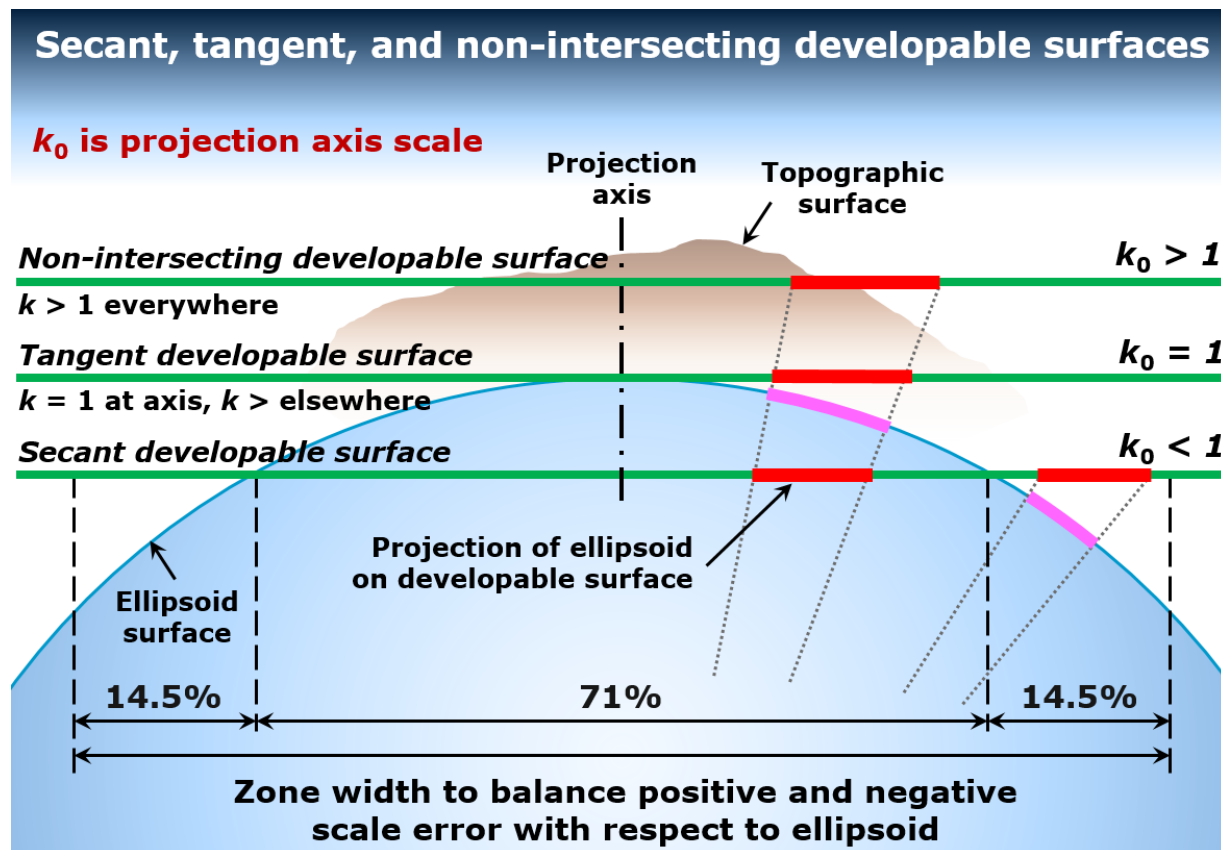


Figure 2.1.2 Secant, tangent, and non-intersection projection developable surfaces.

## 2.1.2 - Map projection distortion

Map projection distortion is an *unavoidable* consequence of attempting to represent a curved surface on a flat surface. It can be thought of as a change in the “true” relationship between points located on the surface of the Earth and the *representation* of their relationship on a plane. Distortion cannot be eliminated — it is a *Fact of Life*. The best we can do is decrease the effect.

There are two general types of map projection distortion, *linear* and *angular*:

1. **Linear distortion.** Although formally defined infinitesimally at a point, it can be thought of as the finite difference in distance between a pair of grid (map) coordinates when compared to the true horizontal (“ground”) distance, denoted here by  $\delta$ .
  - Can express as a ratio of distortion length to ground length:
    - E.g., feet of distortion per mile; parts per million (= mm per km)
    - *Note:* 1 foot / mile  $\approx$  189 ppm = 189 mm / km
  - Linear distortion can be positive or negative:
    - POSITIVE distortion means the grid (map) length is LONGER than the “true” horizontal (ground) length.



- NEGATIVE distortion means the grid (map) length is SHORTER than the “true” horizontal (ground) length.
2. Angular distortion. For conformal projections, it equals the *convergence (mapping) angle,  $\gamma$* . The convergence angle is the difference between projected grid (map) north and true (geodetic) north – a useful property for surveying applications.
- Convergence angle is zero on the projection central meridian, positive east of the central meridian, and negative west of the central meridian. While this is strictly true for the LCC and TM projections, it is not quite the case for the OM (see the next item).
  - For the OM projection, the convergence angle is not zero everywhere along the meridian passing through its local origin (its “central meridian”). The convergence angle is exactly zero only at the local origin. It departs from zero slowly along the central meridian, and in most cases differs from zero by less than 5 arc-minutes within 500 km north and south of the local origin.
  - Magnitude of the convergence angle increases with distance from the central meridian, and its rate of change increases with increasing latitude, as shown in Table 2.1.2. This table can also be used to estimate the convergence angle for the OM east and west of the latitude of the local origin.
  - Usually convergence is not as much of a concern as linear distortion, and it can only be minimized by staying close to the projection central meridian (or limiting surveying and mapping activities to equatorial regions of the Earth). Note that the convergence angle is zero everywhere for the regular Mercator projection, but this projection is not suitable for large-scale mapping in non-equatorial regions due to extreme linear distortion.

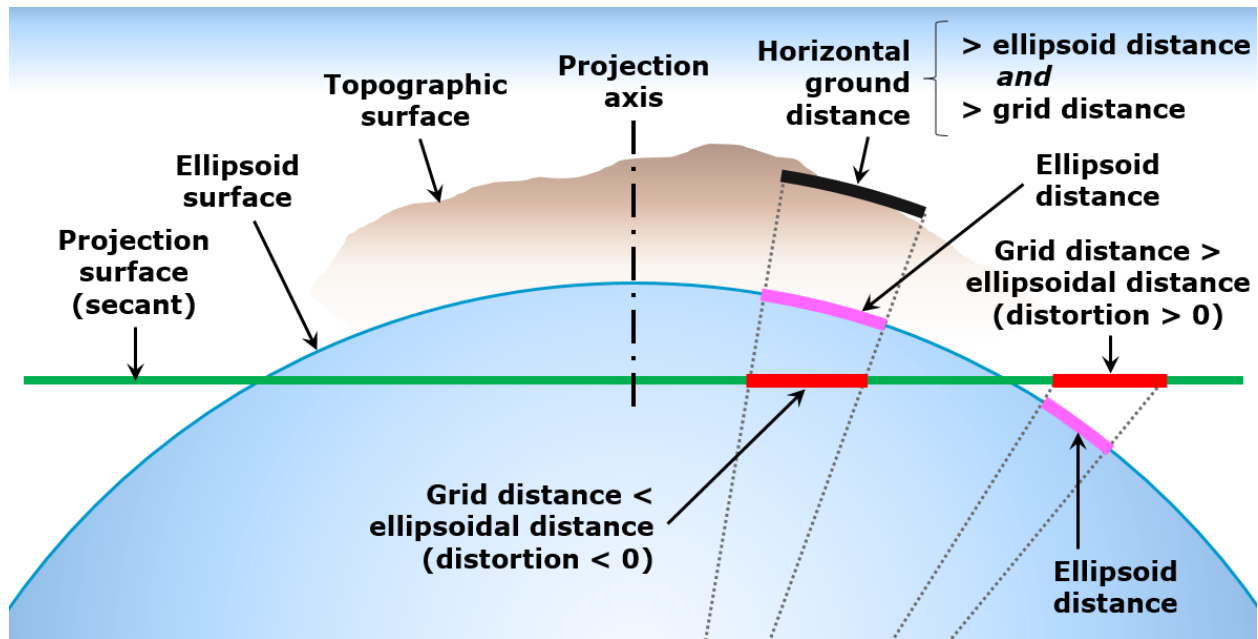
**Table 2.1.2** *Convergence angles at various latitudes, at a distance of one mile (1.6 km) east (positive) and west (negative) of central meridian for TM and projection (and LCC projection with central parallel equal to latitude in table).*

Latitude	Convergence 1 mi from CM	Latitude	Convergence 1 mi from CM	Latitude	Convergence 1 mi from CM
0°	0° 00' 00"	30°	±0° 00' 30"	60°	±0° 01' 30"
5°	±0° 00' 05"	35°	±0° 00' 36"	65°	±0° 01' 51"
10°	±0° 00' 09"	40°	±0° 00' 44"	70°	±0° 02' 23"
15°	±0° 00' 14"	45°	±0° 00' 52"	75°	±0° 03' 14"
20°	±0° 00' 19"	50°	±0° 01' 02"	80°	±0° 04' 54"
25°	±0° 00' 24"	55°	±0° 01' 14"	85°	±0° 09' 53"

One can think of linear distortion as due to the projection “developable surface” (plane, cone, or cylinder) departing from the reference ellipsoid. Although no ellipsoidal forms of conformal projections are perspective (i.e., cannot be created geometrically), it is still useful to think of linear distortion increasing as the “distance” of the developable surface from the ellipsoid increases. In that sense, linear distortion is entirely a function of “height” with respect to the ellipsoid.

Although total linear distortion is (conceptually) due to departure of the developable surface from the ellipsoid, it is convenient to separate it into two components: one due to Earth curvature and one due to height above or below the reference ellipsoid. Indeed, this “total” distortion is often computed as the product of these two components and called the “combined” scale error (or factor). The relative magnitude of each at a point of interest depends on its horizontal distance perpendicular from the projection axis and its ellipsoid height.

Figure 2.1.3 provides a conceptual illustration of distortion as a geometric departure of the developable surface from the reference ellipsoid. Table 2.1.3 gives the range of distortion due to curvature for various projection zone widths, and Table 2.1.4 gives change in distortion due to change in height, but total distortion is always a combination of both.



**Figure 2.1.3** Linear distortion of secant map projection with respect to ellipsoid and topography.

**Table 2.1.3** Maximum range in linear distortion due to Earth curvature for various zone widths (perpendicular to projection axis).

Zone width for secant projections (i.e., balanced positive and negative distortion)	Maximum range in linear distortion, $\Delta(\delta + 1) = \Delta k$		
	Parts per million (mm/km)	Feet per mile	Ratio (absolute value)
16 miles (25 km)	±1 ppm	±0.005 ft/mile	1 : 1,000,000
35 miles (57 km)	±5 ppm	±0.026 ft/mile	1 : 200,000
50 miles (81 km)	±10 ppm	±0.053 ft/mile	1 : 100,000
<b>71 miles (114 km)</b>	<b>±20 ppm</b>	<b>±0.11 ft/mile</b>	<b>1 : 50,000</b>
112 miles (180 km)	±50 ppm	±0.26 ft/mile	1 : 20,000
~158 miles (255 km) e.g., SPCS*	±100 ppm	±0.53 ft/mile	1 : 10,000
~317 miles (510 km) e.g., UTM†	±400 ppm	±2.11 ft/mile	1 : 2500

\*State Plane Coordinate System; †Universal Transverse Mercator

**Table 2.1.4** Change in projection linear distortion due to change in ellipsoid height.

Change in ellipsoid height, $\Delta h$	Change in linear distortion, $\Delta(\delta + 1) = R_G / (R_G + \Delta h)$		
	Parts per million (mm/km)	Feet per mile	Ratio (absolute value)
±100 feet (30 m)	±4.8 ppm	±0.025 ft/mile	~1 : 209,000
<b>±400 feet (120 m)</b>	<b>±19 ppm</b>	<b>±0.10 ft/mile</b>	<b>~1 : 52,000</b>
±1000 feet (300 m)	±48 ppm	±0.25 ft/mile	~1 : 21,000
+2000 feet (610 m)*	-96 ppm	-0.51 ft/mile	~1 : 10,500
+3300 feet (1000 m)**	-158 ppm	-0.83 ft/mile	~1 : 6300
+14,400 feet (4400 m)†	-690 ppm	-3.6 ft/mile	~1 : 1450

\*Approximate mean topographic height of North America

\*\* Approximate mean topographic height in CONUS west of 100°W longitude

† Approximate maximum topographic height in coterminous US

## 2.2 - SPCS Design Techniques

The design approach for SPCS adopted by USC&GS for NAD 27 and subsequently NGS for NAD 83 was relatively simple and straightforward. The *principal* design goal limited mapping distortions relative to the reference *ellipsoid* to one part in ten-thousand (1:10,000, or 100 ppm). At the time of original development of the SPCS the practical ability to design and analyze mapping projections relative to the topographic surface did not exist even though USGS had published the 30-minute topographic quadrangle series at a scale of 1:125,000 (one inch equals approximately two miles). On a more fundamental level, the classical techniques implemented on the ground for the typical survey at that time (i.e. transit and chain or steel tape) generally did not result in accuracies exceeding the 1:10,000 threshold chosen for SPCS, particularly in rough and challenging terrain common in Kentucky. Simply put, the SPCS was originally designed based on practical considerations and mathematical conveniences that were quite reasonable given the general state of surveying and mapping practices at that time.

As can be seen in Table 2.1.3 and as illustrated in Figure 2.1.2, this limited SPCS zones to a maximum width of approximately 158 miles (255 km) as measured perpendicular to the projection axis. This limitation resulted in Kentucky being divided into two zones, North and South, based on the Lambert conformal conic projection method, due to the Commonwealth having much greater extent in the east-west direction (approx. 425 miles) compared with its north-to-south extent (approx. 180 miles), with the two zones similarly following suit. Having said that, this arrangement worked very well for Kentucky, inasmuch as it was utilized, for several decades following its initial implementation given a compatibility with the surveying and mapping capabilities and needs of that era. After all, the initial build-out of the Interstate Highway System was designed and constructed based on the NAD 27 version of the SPCS. However, as presented in the history section of this document, that compatibility eventually

and thereafter quickly diminished as technological advancements made those original assumptions and practicalities functionally obsolete.

Leading up to and during the period in which the KY1Z projection was developed and adopted, the need for SPCS zones to be optimized on the topographic surface had become unavoidable. In the years following adoption of the KY1Z, several states had independently come to the same conclusion and subsequently developed their own projected coordinate reference systems, albeit outside the purview of the national SPCS. These systems were based on highly refined low distortion projection (LDP) techniques developed in response to the obvious need for this approach. Kentucky may have been the first state in the union to establish an SPCS zone predicated on minimizing projection distortions between the ground and grid through projection optimization now associated with LDP techniques. However, the methodologies used to derive the final KY1Z configuration were neither widely known nor widely published outside of Kentucky for mainstream consumption. The KY1Z was simply developed, adopted, and quietly—in a national sense—implemented as a matter of routine. It was primarily through the work of others applying and documenting LDP techniques for various states (and at least one tribal authority) that the practice of developing coordinate reference systems achieving LDP performance relative to the ground would form the basis for NGS adopting ground-to-grid optimization as *the* standard for establishing future versions of the SPCS.

### 2.3 - Ground to Grid Optimization

Ground to grid optimization is not a new concept and its implementation over time has taken various forms. Early attempts include the state of Michigan redefining its SPCS on NAD 27 in 1964 by changing the projection type of its original three zones from TM to LCC and essentially creating a new datum by scaling the Clarke 1866 ellipsoid to emulate an average elevation of 800 feet for the state. This was accompanied by limiting the magnitude of statewide mapping distortions computed between the *conceptualized* topographic surface and the projection grid to 1:10,000 (Dennis, 2018, p. 6). Of particular note regarding this approach is the continued use of 1:10,000 as an acceptable limit for mapping distortions, even between the ground and grid, indicating a commensurate compatibility with surveying methods and technologies still available at the time. The interesting choice of adopting LCC over TM and scaling the reference ellipsoid to fit an average topographic elevation also gives testament to the powerful lure and practical affinity for accepted mathematical conveniences, using the 14.5–71.0–14.5 percent coverage rule (shown in Figure 2.1.2) over an adjusted allowable projection width limited to a 1:10,000 distortion ratio for *secant* projections. It is notable that this approach was adopted during an era when applying the rigorous mathematical procedures required to define SPCS projections remained manually arduous and resource intensive.

Other attempts at compensating for the ground to grid distortion conundrum involved scaling (usually up) state plane coordinates through the inclusion of an average elevation factor as determined for a given area of interest. While this approach is covered in sufficient detail within *NOAA SP NOS NGS 13* (Dennis, 2018, p. 24) one such attempt implemented by the Kentucky Transportation Cabinet (KYTC) through *Design Memorandum No. 5-05* (KYTC, 2005) involved “scaling” the state plane mapping grid to achieve “project datum coordinates”, or a PDCS, that emulated ground distances generally for a given project. The procedure outlined in

*KYTC DM5-05* involved 1) determining the normal mapping scale factor (SF) based on SPCS proper as computed at the central most control monument established for a given project; 2) computing an elevation factor (EF) based on the average elevation for the entire project area as applied at the central control monument; 3) computing a combined factor (CF) as the product of the SF and the EF ( $CF = SF \times EF$ ); and finally 4) computing the project datum factor (PDF) as the reciprocal of the combined factor ( $PDF = 1/CF$ ). New project coordinates were then established by multiplying the normal state plane coordinate values for all project control monuments by the PDF.

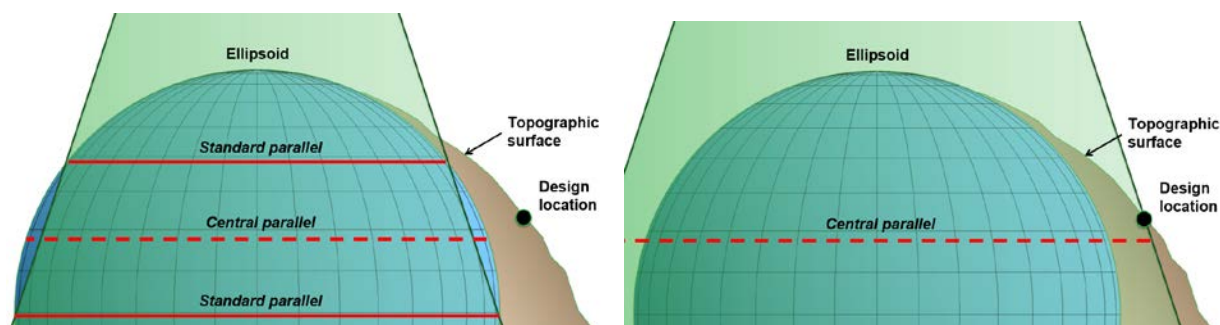
While the overall concept of scaling state plane coordinates to generally fit the ground for a given area of interest appears sound in theory, its implementation as a practical matter came with its own set of complications, most notably the practice of not dropping a strategic number of leftmost digits from the project datum coordinate values. Leaving the full set of digits resulted in the two systems (SPCS vs PDCS) often being indistinguishable from one another. In addition to this general complication, *KYTC Design Memorandum No. 6B-04* (KYTC, 2005) declared in part that “All right-of-way monumentation shall be defined by the Kentucky State Plane Coordinate System, using project datum coordinates...” and “A project datum factor that relates the State Plane Coordinates and project datum coordinates for the right-of-way shall be published on the coordinate control sheets in the design plans.” This casual intermingling of SPCS with project datum coordinates and the lack of pertinent metadata being provided beyond the purview of the highway design plans proved to be a prime source of confusion as to what a coordinate pair associated with a KYTC highway project actually represented (a condition intolerable to land surveyors when encountering KYTC right-of-way documentation established under this policy during their retracement work).

This condition eventually led to the abandonment of the PDF approach through *Design Memorandum No. 05-09* (KYTC, 2009) in which the unmodified Kentucky Single Zone (KY1Z) was adopted as the official coordinate system for all *new* KYTC projects established thereafter. The memorandum stated that “The Project Datum Factor, as it related to State Plane Coordinates and Project Datum Coordinates that was a requirement for KYTC projects, is not needed when utilizing the Kentucky Single Zone System”. Ten years on *KYTC DM05-09* still stands as official KYTC policy.

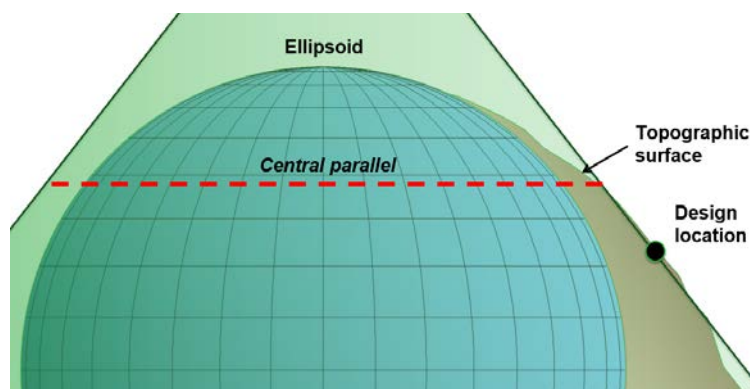
In addition to the above described complications, attempting to reduce distortions by either scaling the ellipsoid (*scenario a*) or an existing projection plane (*scenario b*) to “fit” the ground may result in unforeseen complications or not even accomplish the goal of minimizing distortions over a given target area due to the following reasons (Dennis, 2019):

- Requires a new ellipsoid for every coordinate system. Therefore, the five or six projection parameters plus two ellipsoid parameters are required, for a total of seven or eight parameters to define each system (*scenario a*).
- Coordinates must be transformed to the new ellipsoidal system prior to being projected. So, projection algorithm must include a datum transformation, and this can make these systems more difficult to implement (*scenario a*).

- The transformed latitudes of points can differ substantially from the original values, by more than 3 feet (approx. 1 meter) for heights greater than 1000 ft (300 m). This can cause incorrect projected coordinates if original geodetic coordinates are not transformed prior to projecting (*scenario a*).
- Creates a situation where the projection is strictly only conformal with respect to the “scaled” ellipsoid, rather than the reference ellipsoid. Although such departure from conformality is small, it nonetheless is undesirable, since the entire purpose of using a conformal projection is to maintain conformality with respect to the geodetic datum, as represented by the original (unscaled) reference ellipsoid (*scenario a*).
- Extent of low-distortion coverage generally *decreases* as distance from projection axis increases (*scenario b*).
- State Plane axis usually does **not** pass through the project area and in addition may be oriented in a direction that decreases the area of low distortion coverage (*scenario b*).
- Figures 2.3.1 and 2.3.2 illustrates this problem with “modified” SPCS as compared to an LDP.



**Figure 2.3.1** Typical SPCS situation (for LCC projection) on left. Projection is secant to ellipsoid, with developable surface below topographic surface. Figure on the right depicts a situation in which SPCS is scaled “to ground” at design location. Central parallel in same location as original SPCS; note developable surface inclined with respect to topographic surface. (Dennis, 2019).



**Figure 2.3.2** LDP design. Note central parallel moved north to align developable surface with topographic surface, thereby reducing distortion over a larger region (Dennis 2019).

## 2.4 - Projection Optimization and Low Distortion Design

In the general sense, projection *optimization* refers to achieving optimal performance for a projection, as evaluated between the topographic surface and projection grid, by minimizing and balancing the magnitude of distortions experienced across the entire projection area of interest, *regardless of its size*. Variations on this approach include skewing the projection axis to favor a portion of the overall target area, such as an urban area or region of high value or special use, while maintaining a desired maximum allowable distortion threshold, albeit perhaps not the optimal threshold achievable when the projection is generally centered on the area of interest and completely balanced throughout. This approach was implemented during the design phase of the original *Kentucky Single Zone Coordinate System of 1983 (KY1Z-1983)* to slightly favor the urbanized areas in northernmost Kentucky by skewing the standard parallels northward while maintaining ground to grid distortions experienced in far southwestern Kentucky to less than 200 parts per million (within 1:5,000).

The primary goal of achieving low distortion performance is to optimize a projection relative to the topographic surface so as to achieve a pre-defined maximum allowable magnitude of distortion small enough to be reasonably ignored for most, if not all, intended purposes. Linear distortion is expressed here as lower-case delta ( $\delta$ ) in parts per million (ppm) within statistically defined constraints (e.g.,  $\pm 20$  ppm for 50% of the total coverage area). For the purposes of this document, the term low distortion projection (LDP) will be based on NOAA NGS 2019-1214-01-A2 Procedures for Design and Modification of the State Plane Coordinate System of 2022 (NGS, 2019, p. 10) and defined as follows:

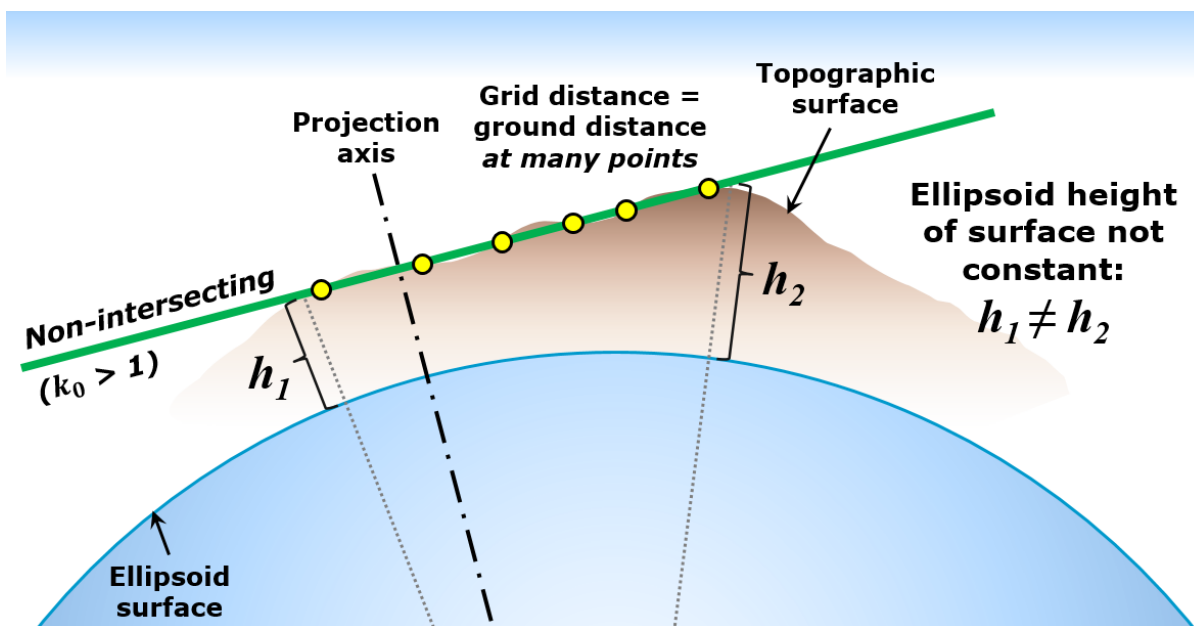
- The linear distortion design criterion achieves less than  $\pm 50$  ppm and satisfies all three minimum percentages:
- 90% of zone population.
- 75% of cities and towns (based on location only, irrespective of population).
- 50% of total zone area.

In Figure 2.1.2, the relationships between the projection surface, reference ellipsoid, and topographic surface are shown for given scenarios for the defining scale factor  $k_0$ . For Kentucky the topographic surface is *always* above the ellipsoid (except in the isolated case of a quarry located in the far western portion of the state in which the bottom of the pit extends below vertical datum), and the surface of the geoid is *always* below the ellipsoid, as it is for CONUS. Under these circumstances it will *generally* be the case for low distortion performance that a projection defined with  $k_0$  being greater than or equal to one will result in optimal performance, given it pushes the projection surface outward toward the terrain.

For regions in which the topographic surface varies considerably in a manner that results in a significant regional slope trend, adjusting  $k_0$  upward alone may not be enough to achieve projection optimization, particularly when the design scope includes a desire to maximize the spatial domain of a given projection. In special cases where the regional slope trend runs generally perpendicular to the projection axis it is possible to “tilt” the projection axis so as to bring the projection surface (which is fixed orthogonally to the projection axis) in line with the

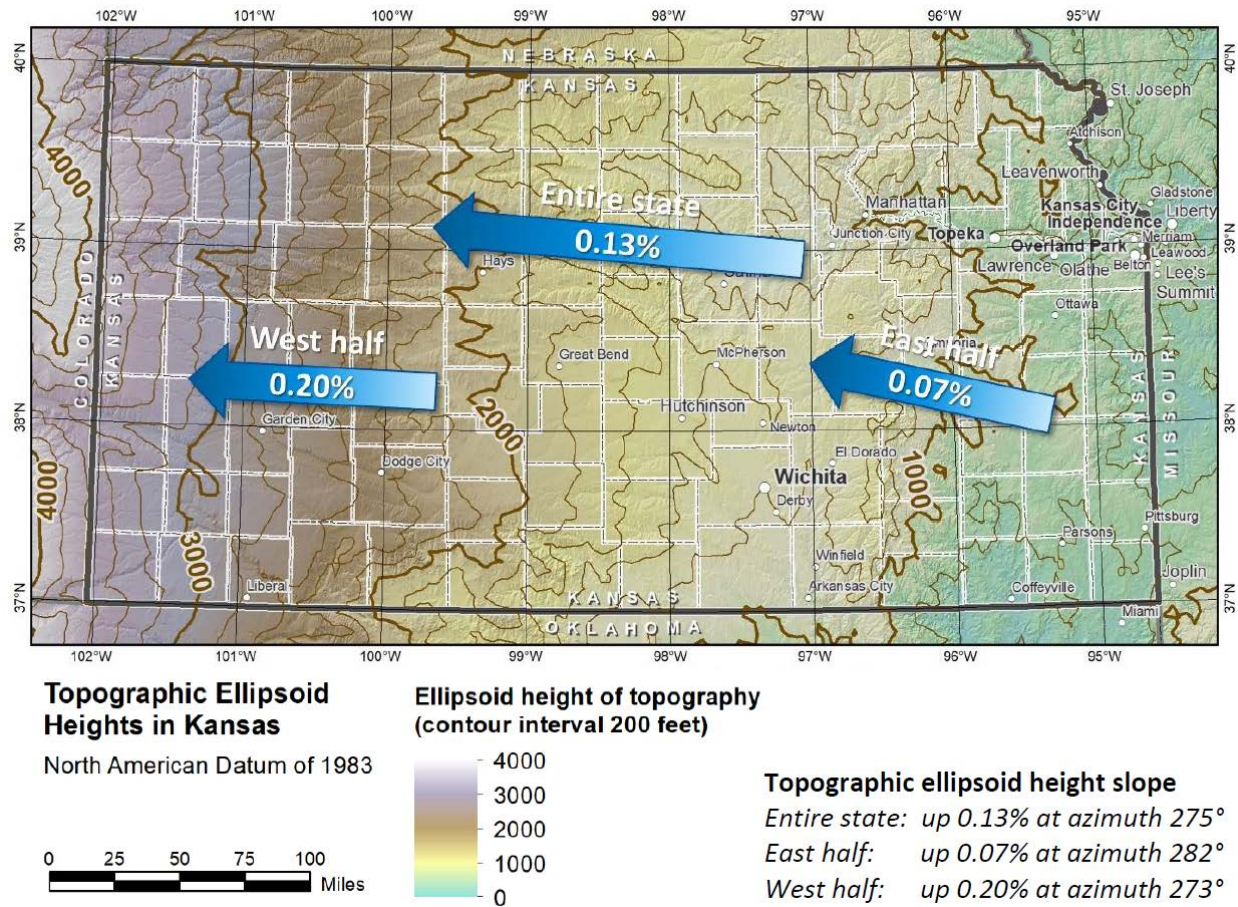


terrain of the targeted projection area (Fig. 2.4.1). This technique can be quite effective in minimizing ground to grid distortions. For TM and OM projections, offsetting the projection axis will generally increase the range in convergence angles in the design area, although the increase is usually not large enough to be considered a problem (for LCC projection axis offsets, there is no effect at all on the convergence angles).



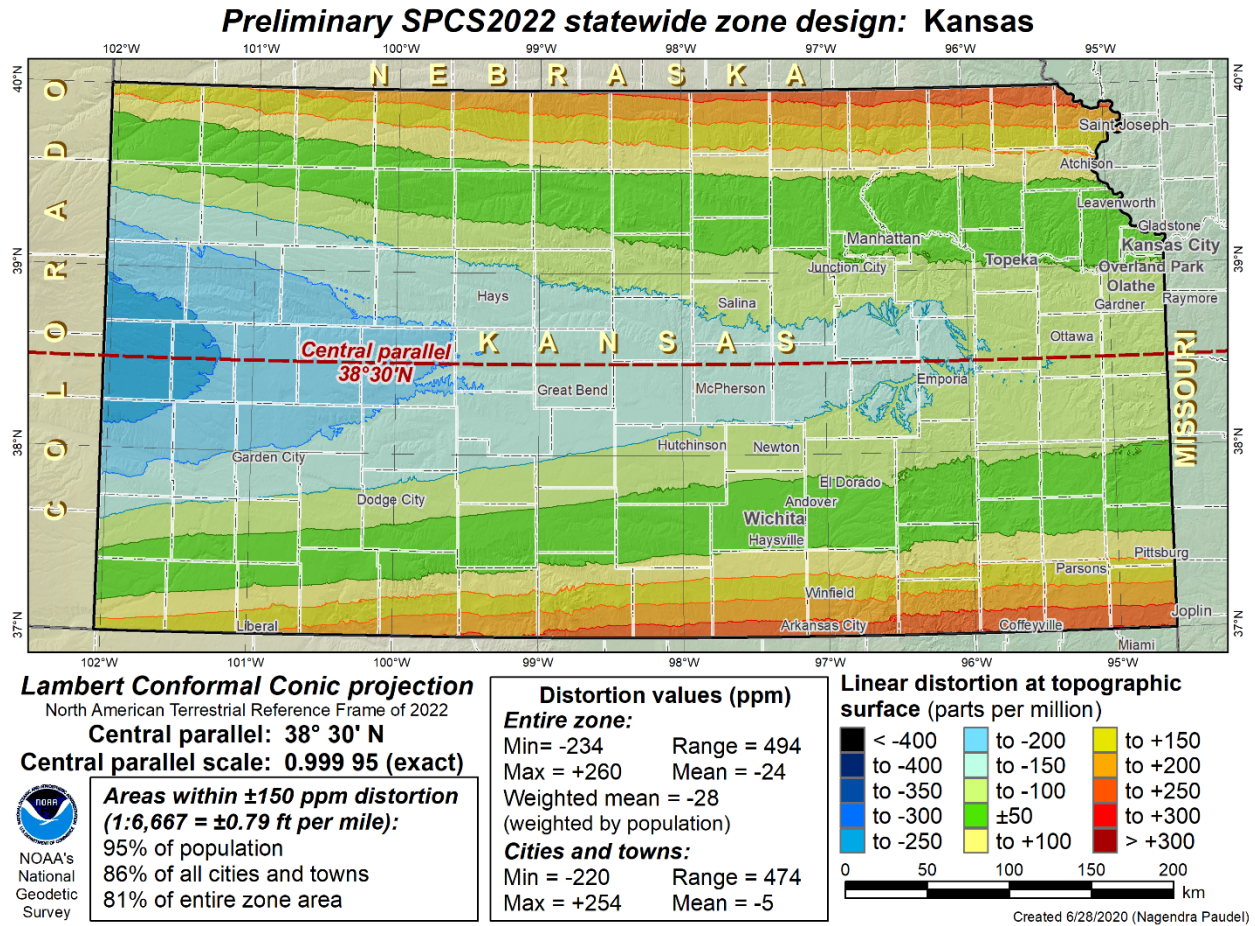
**Figure 2.4.1** Tilting the projection axis to bring the projection surface in line with the terrain. (Dennis, 2019).

Conversely there will be cases where regional topographic trends are not amenable to achieving projection optimization through the above described technique, particularly for larger statewide zones where the choice of projection type is limited. One classic example of this scenario is the state of Kansas, which is quite similar to Kentucky in that its minimum bounding box results in a wide rectangle oriented in the east-west direction, being well suited for the LCC projection but has a regional topographic surface resulting in a relatively flat, but significantly sloping trend running parallel to the LCC projection axis (Fig. 2.4.2). Under these conditions projection optimization is limited to adjusting  $k_0$  to best fit the topographic surface while keeping the projection axis generally centered north to south (tilting the projection axis does not help in this case).



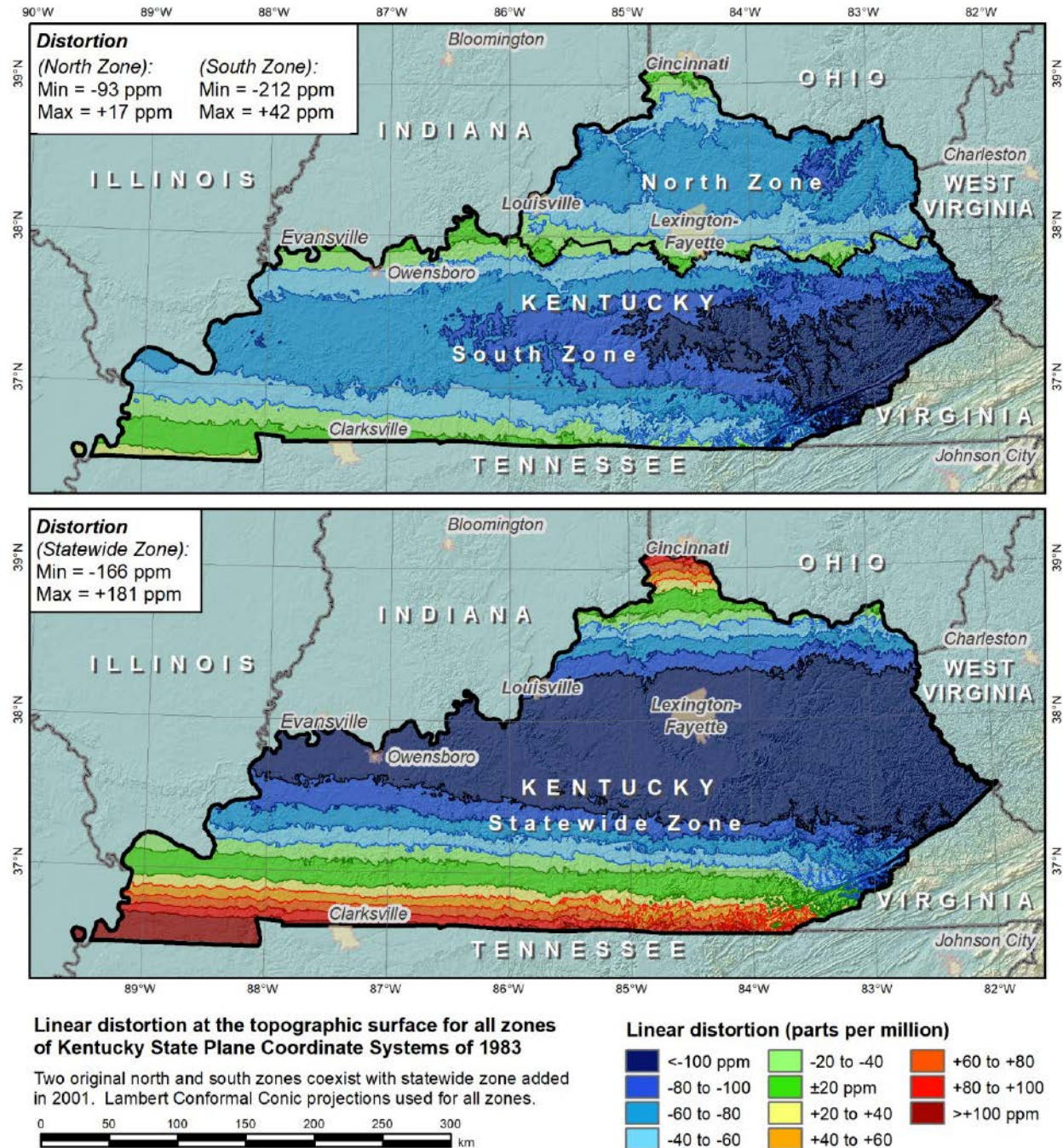
**Figure 2.4.2** Topographic ellipsoid height and associated sloping trends in Kansas. (Dennis, KDOT, 2017)

Figure 2.4.3 shows a distortion map for the preliminary SPCS2022 statewide zone design in which the projection axis is oriented parallel to the sloping trend causing the orthogonally fixed projection surface to diverge from surface topography, thus resulting in a “V” shaped pattern pointing toward the downward sloping trend, particularly for the west half of the state. For the LCC projection this pattern points east or west (as can be seen below) and for the TM projection this pattern will be conversely oriented north or south. Had the topographic surface been amenable to locating the projection axis such that the projection surface generally emulated the topographic surface then the colored bands would have run nearly parallel to each other throughout (as in the easternmost quarter of the state), influenced mainly by the curvature of the reference ellipsoid.



**Figure 2.4.3** Linear distortion for the preliminary SPCS2022 statewide Kansas zone design, in which the projection axis runs parallel to a regional sloping trend. Note the “V” shaped pattern pointing towards the downward sloping trend. (Dennis, 2020).

A similar LCC pattern for Kentucky is evident in Fig 2.4.4 below, albeit not nearly as pronounced and in the reverse direction. Of particular note are the differences between the original NAD 83 North and South zones, designed to *limit* distortions relative to the reference ellipsoid to 1 part in 10,000, and that of the KY1Z, designed to *minimize* distortions relative to the topographic surface throughout the state (with a minor tilting northward to favor the urbanized areas of northernmost Kentucky). As can be seen, with the exception of the northern and southern flanks for each of the North and South zones, the projection distortions are predominantly negative indicating a projection surface lying below the surface terrain. This is to be expected given the 14.5 – 71.0 – 14.5 percent coverage rule for defining and balancing projections relative to the reference ellipsoid, meaning 71% of the projection surface is *below* the ellipsoid, which for Kentucky translates into *most* of the projection surface being below the topographic surface within the respectively defined regions. Conversely, because the KY1Z was designed to optimize ground to grid performance throughout its intended area of interest the resulting distribution of distortion is more balanced between positive and negative values and in magnitude.



**Figure 2.4.4** Linear distortion at the topographic surface for the NAD 83 SPCS zones in Kentucky: the original north and south zones (top) and the statewide KY1Z added in 2001 (bottom). (Dennis, 2018)

## Chapter 3: Policy and Standards

### 3.1 - Conformance with NGS Policy

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*National Geodetic Survey*

The policy document issued by NGS on the State Plane Coordinate System (SPCS) of 2022 is identified as Tracking Number 2019-1214-02, herein referenced as the SPCS2022 Policy, and was created by NGS as the agency being the original creator and steward of the SPCS. As the creator of this system, NGS has sole authority in defining any policies regarding design of SPCS zones. As of this writing there are two editions of the SPCS2022 Policy, with the second one listed above being the edition issued in October 2019, and with a planned review schedule of two year intervals.

This Policy was developed and published to provide some *general information* on: background/history, underlying geodetic characteristics of SPCS2022, allowable zone definitions, multi-layering of zones, and units of measure. Separate from the Policy but closely tied to it is also the SPCS2022 Procedures (Tracking Number 2019-1214-01-A2). Although these two documents go “hand-in-hand” in many ways, it is important to make the distinction that the Procedures document *outlines technical information* such as: design details and restrictions, submittal POCs and deadlines, naming conventions, parameter restrictions, requests for exceptions, and the like. The overall difference between the Policy and Procedures documents, from a bureaucratic standpoint, is that NGS policy is required to be approved by the Executive Steering Committee or the Director, while procedures can be modified and reissued with lower level authorization.

The NGS SPCS2022 Policy document is available here:

<https://geodesy.noaa.gov/INFO/Policy/files/SPCS2022-Policy.pdf>

The NGS SPCS2022 Procedures document is available here:

<https://geodesy.noaa.gov/INFO/Policy/files/SPCS2022-Procedures.pdf>

### 3.2 - Meter to Customary Foot Conversion Factors

Linear units of measure are obviously a crucial element in geodesy and accurately defining conversion factors between various systems of linear measurement is paramount to achieving consistent, repeatable, and reliable results when such conversions are required. In deriving the Clarke 1866 ellipsoid the process of establishing conversion factors between the various systems of linear measure in use at the time to a very high degree of accuracy consumed the vast majority of the work through which the parameters defining the “Spheroid of Rotation” were derived. The resulting publication was titled *Comparisons of the Standards of Length of England, France, Belgium, Prussia, Russia, India, Australia, Made at the Ordnance Survey Office, Southampton, by Captain A. R. Clarke, R.E., F.R.S., Under the Direction of Sir Henry James, R.E.,*

*F.R.S. & c.* and of the 287 numbered pages contained within only the last 7 were dedicated to the determination of the “Figure of the Earth”, with the preceding 280 numbered pages being dedicated to the scientific rigors of comparing standard rods for the various units of linear measure implemented during the triangulation surveys included in the analysis presented in those last seven pages. In determining a conversion factor between the toise [twaz], a standard unit of length of approximately two meters (6 French feet), and the English foot, the greatest difference between independent results involving the Russian standard, Prussian standard, and the Belgian standard totaled half a millionth toise, “a difference corresponding to ten feet in the earth’s radius”.

Whether by coincidence or not, the meter was declared legal as a unit of measure by Congress in 1866, and by 1893 it became the basis of linear measure and for defining the foot for the USC&GS through the Mendenhall Order, as given by its then superintendent, Thomas Mendenhall. Through that declaration the foot was defined as being 1200/3937 of a meter, an integer quotient resulting in a repeating decimal inverse of 3.28083333333... While this familiar factor is now recognized as the United States survey foot (USFt), it was not identified as such until the National Bureau of Standards (NBS), predecessor to the National Institute of Standards and Technology (NIST), adopted the International foot (previously declared in 1933 by NBS as 0.3048 meter, exactly, and subsequently adopted by the predecessor to NASA in 1952) as the standard in 1959, with the exception of allowing continued use of the USFt for geodetic surveying activities through the following 1959 Federal Register declaration:

“Any data expressed in feet derived from and published as a result of geodetic surveys within the United States will continue to bear the following relationship as defined in 1893:

$$1 \text{ foot} = 1200/3937 \text{ meter}$$

The foot unit defined by this equation shall be referred to as the U.S. Survey Foot and it shall continue to be used, for the purpose given herein, **until such a time as it becomes desirable and expedient to readjust the basic geodetic survey networks in the United States, after which the ratio of a yard, equal to 0.9144 meter, shall apply.** (emphasis added).

Thus, by 1960 there were two definitions applicable to the customary foot in use, albeit a presumed temporary condition, but differing between them by 2 ppm. By 1989 when NAD 83 had been announced, the 1959 FRN caveat declaring temporary usage of the USFt had been effectively ignored resulting in its continued use within the SPCS as defined on the NAD 83 datum by various states (in 1977 NGS itself had gone completely metric, thus relegating definition of the customary foot relative to the meter to the individual states). Given the general momentum of legacy customs and traditions, the Kentucky legislature defined the USFt as the legal unit of linear measure for expressing coordinate values on the Kentucky Plane Coordinate System of 1983 through its 1992 enactment of KRS 1:020.

While this approach achieved consistency within the Commonwealth with respect to the SPCS as it was defined on NAD 27 and NAD 83 (even though the SPCS as defined on NAD 27 had not been adopted by statute), the lack of a declared national standard for defining the customary foot resulted in a choice between two options when defining the foot for SPCS projected coordinate systems within surveying and GIS software, a condition for users either not paying attention to coordinate system settings or not aware of the differences between those particular two options could result in positions described by SPCS coordinates not being properly referenced and in error by several “feet” given the magnitude in range of SPCS coordinate values. This is an important consideration because at its core the SPCS is a rigorous mathematical construct and reliance on automated and integrated computational capabilities and functionality is crucial to its continued implementation simply as a practical matter.

As of July, 2020 the Commonwealth has revised its statutory framework associated with the KSPCS such that future implementations will adopt what is currently identified as the International foot. Statutory changes made during the January 2020 session of the Kentucky legislature has incorporated the following language within Section 3 of KRS 1:020 for defining how the customary foot as it relates to the meter will be defined relative to a given series (datum):

**KRS 1:020 (3)** ...Unless otherwise originally established for an existing series the base unit of linear measure for defining all zones within each series of the KSPCS shall be the meter. The specific constant for converting distances within each zone from the meter to the customary foot shall be (a) the U. S. survey foot conversion factor as originally and exclusively specified for any existing series, and (b) the International foot conversion factor exclusively for each subsequent series established hereafter.

The above language has been adopted not only in anticipation of future policy signaled by NGS for SPCS on NATRF2022, but also by NIST through FRN 84 FR 55562 in which it declares, in part, the following:

On December 31, 2022, the 1893 “U.S. survey foot,” as defined in a 1959 Federal Register notice (24 FR 5348, June 30, 1959), will be deprecated as a U.S. national standard of measurement and its use is to be avoided. The 1893 definition of the “U.S. survey foot” will be retained for historic reference but will be deemed obsolete.

Thus, as provided in the proposed revision to KRS 1:020 (3), the definition of the customary foot relative to the meter shall be the legacy U.S. survey foot as originally defined 1) by NGS for SPCS 27, and 2) the Kentucky legislature through the 1992 enactment of KRS 1:020 for NAD 83, and for all future versions of the KSPCS, including NATRF2022, the customary foot will be defined by what is currently identified as the International foot as adopted by NIST through 24 FR 5348, June 30, 1959.

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## Appendix A: Kentucky Revised Statutes

### A.1 - 1.010 Legislative intent in establishing Kentucky State Plane Coordinate System.

It is the intent of the General Assembly of the Commonwealth of Kentucky that KRS 1.020 shall not eliminate the existing methods of describing points on, within, or above the surface of the earth, as in metes and bounds or, in western Kentucky, the public land system, but rather to enhance these existing methods and establish a conformity for defining and stating the geographic positions or locations of points on, within, or above the surface of the earth and retracement purposes.

Effective: July 15, 2020

History: Amended 2020 Ky. Acts ch. 77, sec. 1, effective July 15, 2020. –Created 1992 Ky. Acts ch. 419, sec. 2, effective July 14, 1992.

### A.2 - 1.020 Kentucky State Plane Coordinate System.

- (1) The Kentucky State Plane Coordinate System, which is hereby adopted, means a system of plane coordinates which have been established by the National Oceanic and Atmospheric Administration, through its National Geodetic Survey, for defining and stating the geographic positions or locations of points on, within, or above the surface of the earth within the Commonwealth of Kentucky.
- (2) For this system, the Commonwealth, through the Commonwealth Office of Technology, under KRS 42.650 and advised by the Geographic Information Advisory Council, under KRS 42.740, shall establish and publish a series of layered zones covered by geodetically referenced mapping projections adopted and supported by the National Geodetic Survey as a component of the National Spatial Reference System. Each series of zones shall be identified by the geodetic datum upon which they are defined and each zone shall remain uniquely and consistently defined throughout its implementation within a particular series.
- (3) One (1) U. S. survey foot equals  $(1200)/(3937)$  meter. For conversion of meters to U. S. survey feet, multiply the meters by 3.280833333333 to twelve (12) significant figures. One (1) international foot equals 0.3048 meter exactly. For conversion of meters to international feet, multiply the meters by 3.280839895. Unless otherwise originally established for an existing series, the base unit of linear measure for defining all zones within each series of the Kentucky State Plane Coordinate System shall be the meter. The specific constant for converting distances within each zone from the meter to the customary foot shall be:
  - (a) The U.S. survey foot conversion factor as originally and exclusively specified for any existing series; and
  - (b) The international foot conversion factor exclusively for each subsequent series established hereafter.

- (4) The plane coordinate values to be used for expressing the geographic position or location of a point in the appropriate zone of the Kentucky State Plane Coordinate System shall consist of two (2) distances expressed in customary feet and decimals of a foot or meters and decimals of a meter. When the values are expressed in customary feet, the meter to foot conversion factor for the respective Kentucky State Plane Coordinate System series, as specified in subsection (3) of this section, shall be used. One (1) of the distances, to be known as the "North y-coordinate," shall give the distance north of the X axis. The other, to be known as the "East x-coordinate," shall give the distance east of the Y axis. The Y axis of any zone shall be parallel with the central meridian of that zone. The X axis of any zone shall be at right angles to the central meridian of that zone.
- (5) For purposes of describing the location of any survey station or land boundary corner in the Commonwealth of Kentucky, it shall be considered a complete, legal, and satisfactory description of the location to give the position of the survey station or land boundary corner on the Kentucky State Plane Coordinate System.
- (6) Nothing contained in this section shall require a purchaser or mortgagee of real property to rely wholly on a land description any part of which depends exclusively upon the Kentucky State Plane Coordinate System.
- (7) When any tract of land to be defined by a single description extends from one (1) into multiple mutually adjacent zones, the position of all points on its boundaries shall be referred exclusively to one (1) of the multiple zones. The zone which is used shall be named in the description.
- (8) No coordinates based on the Kentucky State Plane Coordinate System, purporting to define the position of a corner on a land boundary, shall be presented to be recorded in any public land records or deed records unless the corner has been tied to a control monument or station established by conforming to the standards of accuracy for boundary surveying as specified by administrative regulations duly promulgated under KRS Chapter 322.
- (9) The use of the term "KENTUCKY STATE PLANE COORDINATE SYSTEM" on any map, report of survey, or other document shall be limited to coordinates based on the Kentucky State Plane Coordinate System as defined in this section.
- (10) If any provision of this section or the application thereof to any person or circumstance is held invalid, the invalidity shall not affect other provisions or applications of the section which can be given effect without the invalid provision or application, and to this end the provisions of this section are severable.
- (11) The provisions of this chapter shall not be construed to prohibit the appropriate use of other geodetic reference networks.

**Effective:** July 15, 2020

**History:** Amended 2020 Ky. Acts ch. 77, sec. 2, effective July 15, 2020. -- Created 1992 Ky. Acts ch. 419, sec. 3, effective July 14, 1992.

## Appendix B: Kentucky Administrative Regulations

### B.1 - 200 KAR 041:010. The Kentucky State Plane Coordinate System (Proposed Language – Not Currently Promulgated)

RELATES TO: KRS 1.020, 42.630, 42.650, 42.740

STATUTORY AUTHORITY: KRS 42.650(5)

NECESSITY, FUNCTION, AND CONFORMITY: KRS 42.650(5) authorizes the Commonwealth Office of Technology (COT) to promulgate administrative regulations to implement that statute.

KRS 1.020 (2) requires the Commonwealth Office of Technology (COT) to establish and publish a series of layered zones covered by geodetically referenced mapping projections adopted and supported by the National Geodetic Survey (NGS) as a component of the National Spatial Reference System (NSRS).

#### Section 1. Definitions.

- (1) COT means Commonwealth Office of Technology
- (2) GIAC means Geographic Information Advisory Council
- (3) NOAA means National Oceanic and Atmospheric Administration
- (4) NGS means National Geodetic Survey
- (5) NSRS means National Spatial Reference System
- (6) SPCS means State Plane Coordinate System
- (7) KSPCS means Kentucky State Plane Coordinate System
- (8) Geodetic datum, as referenced herein, means a geometric model representing the earth's size and shape. The mathematical surface of a geodetic datum is an oblate spheroid, called a reference ellipsoid, generally designed to best fit mean sea level either globally or for a stated region. In the context of a geometric framework in which horizontal coordinates are expressed in angular units as latitude and longitude, a geodetic datum is also referred to as a *terrestrial reference frame*, or simply, *reference frame*.
- (9) Geodetically referenced mapping projection means a planar surface mathematically associated with a geodetic datum, or terrestrial reference frame, such that unique positions relative to that datum or terrestrial reference frame can be converted to and from commensurately unique positions on that plane.
- (10) A *state plane zone*, or *zone*, is a geographic region covered by a uniquely defined geodetically referenced mapping projection and generally comprised of a collection of mutually adjacent whole counties such that all included counties lie completely within a given zone. In special cases a zone may partially cover a county or parts of mutually adjacent counties in order to represent a geographic area of specific interest. A zone may cover the Commonwealth either in part or in whole.

- (11) A state plane layer is a collection of one or more zones, all defined on a common geodetic datum or terrestrial reference frame and designed to achieve, in aggregate, a common theme based on similar performance characteristics that may cover the Commonwealth in part or in whole.
- (12) A state plane series is a collection of one or more layers defined on a common and unique geodetic datum or terrestrial reference frame representing a complete implementation of the national State Plane Coordinate System (SPCS) for the Commonwealth on that datum or terrestrial reference frame.
- (13) The KSPCS is the collection of all series applicable to the Commonwealth of Kentucky.
- (14) *Customary foot* refers to the foot as a unit of measure in a generic sense outside the context of a specific conversion regimen.

## Section 2. Incorporation by Reference.

- (1) The Commonwealth Office of Technology (COT), as advised by the Geographic Information Advisory Council (GIAC), shall develop and maintain the *Kentucky State Plane Coordinate System Standards and Specifications Document*, herein referred to as the *KSPCS Standards and Specifications Document*.
- (2) The *KSPCS Standards and Specifications Document* is incorporated by reference.
- (3) The *KSPCS Standards and Specifications Document* may be inspected, copied, or obtained, subject to applicable copyright law at the Commonwealth Office of Technology, Frankfort, Kentucky, Monday through Friday, 8 a.m. to 4:30 p.m. <Provide URL>
- (4) The *KSPCS Standards and Specifications Document* shall provide pertinent information and narratives required to adequately describe implementation of the KSPCS, including historical context, underlying concepts, and policy. Additional information not specifically required herein but deemed necessary to facilitate greater understanding of the KSPCS may also be included within the document.
- (5) The *KSPCS Standards and Specifications Document* shall reconcile or otherwise clarify nomenclature and terminology adopted and/or refined by NGS when such adaptations result in ambiguities relating to similar terms and language utilized within KRS 1:010, KRS 1:020, and/or this KAR.
- (6) For each state plane series adopted, the *KSPCS Standards and Specifications Document* shall provide a detailed description containing:
  - (a) The series name,
  - (b) The datum or terrestrial reference frame upon which the series is defined, including the reference ellipsoid and its defining parameters, and
  - (c) The linear units of measure used to define the series and, when applicable, the forward and reverse conversion factors to be used for converting between the meter and customary foot when representing linear measurements.
- (7) For each layer within a KSPCS series, a detailed description shall be provided containing:

- (a) The name of the layer, and
  - (b) The purpose of the layer.
- (8) For each zone within a KSPCS layer a detailed description shall be provided containing:
- (a) The zone name.
  - (b) The conformal projection type utilized for that zone.
  - (c) The Central Parallel, expressed as degrees and whole minutes of latitude including the North direction indicator from the equator. When implementing the double standard parallel definition of the Lambert Conformal Conic projection type, the North Standard Parallel and South Standard Parallel, both expressed as degrees and whole minutes including the North direction indicator from the equator shall be provided in lieu of the Central Parallel.
  - (d) The Central Meridian, expressed as degrees and whole minutes of longitude including the East or West direction indicator from the prime meridian.
  - (e) When implementing the double standard parallel definition of the Lambert Conformal Conic projection type, the Base Parallel is provided, expressed as degrees and whole minutes of latitude including the North direction indicator from the equator, representing the basis of the false northing and false easting coordinate values for establishing the location of the projected grid origin. For all other projection types, the Central Parallel shall be used as the basis for the false northing and false easting coordinate values for establishing the location of the projected grid origin.
  - (f) When defined by the transverse Mercator (TM) or oblique Mercator (OM) projection types, *or* implementing the single standard parallel definition of the Lambert Conformal Conic projection type, the projection axis scale factor shall be provided and expressed to 6 full decimal places representing the nearest one part per million increment.
  - (g) The False Northing value, including linear units of measure, to be applied on the projection grid at the intersection of the Central Meridian with the Base Parallel or Central Parallel as specified in item (e) of this section.
  - (h) The False Easting value, including linear units of measure, to be applied on the projection grid at the intersection of the Central Meridian with the Base Parallel or Central Parallel as specified in item (e) of this section.
  - (i) When the oblique Mercator conformal projection type is utilized, the Skew Azimuth of the projection axis, as measured clockwise from geodetic north and expressed in whole positive degrees. When expressed as a quadrant measure regardless of direction, the absolute value of the Skew Azimuth shall fall between 5 degrees and 85 degrees inclusively. The Skew Azimuth is defined at the intersection of the Central Meridian and Central Parallel.
  - (j) When the zone represents a portion of the Commonwealth, a list of the whole counties to which the zone shall exclusively apply. When the zone represents statewide coverage then a statement declaring so shall be provided.

## Appendix C: Series Specifications

### C.1 - Series 1: North American Datum of 1927 (NAD 27)

- **Series Name:** Kentucky State Plane Coordinate System of 1927
- **Datum:** North American Datum of 1927
- **Ellipsoid Parameters:**
  - **Ellipsoid Name:** Clarke 1866 Ellipsoid
  - **Radius of Semi-major (equatorial) axis (a):** 6,378,206.4 meters.
  - **Radius of Semi-minor (polar) axis (b):** 6,356,583.8 meters.
  - **Geometric Inverse of Flattening (1/f):** 294.978698214 (derived).
- **Linear Units of Measure:**
  - **Defining Unit:** The foot as defined by the 1893 USC&G *Mendenhall Order*.
  - **Unit for Describing Coordinates:** The foot as defined by the 1893 USC&G *Mendenhall Order*.
  - **Forward Metric Conversion Factor:** 1 foot = 1200/3937 meter (USC&G, 1893)
  - **Reverse Metric Conversion Factor:** 1 meter = 3.280833333333 feet (computed to 12 decimal places).
- **Layer 1 – Kentucky State Plane Coordinate System of 1927**
  - **Layer Name:** Kentucky State Plane Coordinate System of 1927
  - **Purpose of Layer:** Multi-zone statewide coverage originally designed by the USC&G to achieve 1 part in 10,000 (100 ppm) or better distortion performance relative to the defining ellipsoid.
  - **Zone 1: NAD 27 Kentucky North Zone**
    - **Zone Name:** NAD 27 Kentucky North Zone (KYNZ – 1927)
    - **Projection Type:** Lambert Conformal Conic (LCC)
    - **Central Meridian:** 84° 15' W
    - **North Standard Parallel:** 38° 58' N
    - **South Standard Parallel:** 37° 58' N
    - **Base Parallel:** 37° 30' N
    - **False Northing:** 0 ft
    - **False Easting:** 2,000,000 ft
    - **Included Counties:** Anderson, Bath, Boone, Bourbon, Boyd, Bracken, Bullitt, Campbell, Carroll, Carter, Clark, Elliott, Fayette, Fleming, Franklin, Gallatin, Grant, Greenup, Harrison, Henry, Jefferson, Jessamine, Kenton, Lawrence, Lewis, Mason, Menifee, Montgomery, Morgan, Nicholas, Oldham, Owen, Pendleton, Robertson, Rowan, Scott, Shelby, Spencer, Trimble, Woodford



- **Zone 2: NAD 27 Kentucky South Zone**
  - **Zone Name:** NAD 27 Kentucky South Zone (KYSZ – 1927)
  - **Projection Type:** Lambert Conformal Conic (LCC)
  - **Central Meridian:** 85° 45' W
  - **North Standard Parallel:** 37° 56' N
  - **South Standard Parallel:** 36° 44' N
  - **Base Parallel:** 36° 20' N
  - **False Northing:** 0 ft
  - **False Easting:** 2,000,000 ft
  - **Included Counties:** Adair, Allen, Ballard, Barren, Bell, Boyle, Breathitt, Breckinridge, Butler, Caldwell, Calloway, Carlisle, Casey, Christian, Clay, Clinton, Crittenden, Cumberland, Daviess, Edmonson, Estill, Floyd, Fulton, Garrard, Graves, Grayson, Green, Hancock, Hardin, Harlan, Hart, Henderson, Hickman, Hopkins, Jackson, Johnson, Knott, Knox, Larue, Laurel, Lee, Leslie, Letcher, Lincoln, Livingston, Logan, Lyon, Madison, Magoffin, Marion, Marshall, Martin, McCracken, McCreary, McLean, Meade, Mercer, Metcalfe, Monroe, Muhlenberg, Nelson, Ohio, Owsley, Perry, Pike, Powell, Pulaski, Rockcastle, Russell, Simpson, Taylor, Todd, Trigg, Union, Warren, Washington, Wayne, Webster, Whitley, Wolfe

## C.2 - Series 2: North American Datum of 1983 (NAD 83)

- **Series Name:** Kentucky State Plane Coordinate System of 1983
- **Datum:** North American Datum of 1983
- **Ellipsoid Parameters:**
  - Ellipsoid Name: Geodetic Reference System of 1980 (GRS 80)
  - Radius of Semi-major (equatorial) axis ( $a$ ): 6,378,137 meters (exact).
  - Geometric Inverse of Flattening ( $1/f$ ): 298.257222101 (derived).
- **Linear Units of Measure:**
  - **Defining Unit:** meter.
  - **Unit for Describing Coordinates:** U.S. Survey foot (24 FR 5348, Jun. 30, 1959 and KRS 1:020)
  - **Forward Metric Conversion Factor:** 1 foot = 1200/3937 meter
  - **Reverse Metric Conversion Factor:** 1 meter = 3.28083333333333 feet (computed to 12 decimal places).
- **Layer 1 - NAD 83 Kentucky Single Zone**
  - **Layer Name:** Kentucky Single Zone Coordinate System of 1983
  - **Purpose of Layer:** Single zone statewide coverage designed to minimize projection distortions relative to the topographic surface.
  - **Zone 1: NAD 83 Kentucky Single Zone**
    - **Zone Name:** NAD 83 Kentucky Single Zone (KY1Z-1983)
    - **Projection Type:** Lambert Conformal Conic (LCC)
    - **Central Meridian:** 85° 45' W
    - **North Standard Parallel:** 38° 40' N
    - **South Standard Parallel:** 37° 05' N
    - **Base Parallel:** 36° 20' N
    - **False Northing:** 1,000,00 meters
    - **False Easting:** 1,500,000 meters
    - **Included Counties:** Statewide coverage (all counties).

- **Layer 2 – NAD 83 Kentucky North and South Zones**
  - **Layer Name:** NAD 83 Kentucky North and South Zones
  - **Purpose of Layer:** Multi-zone statewide coverage originally designed by NGS to achieve 1 part in 10,000 (100 ppm) or better distortion performance relative to the defining ellipsoid.
  
- **Zone 1: NAD 83 Kentucky North Zone**
  - **Zone Name:** NAD 83 Kentucky North Zone (KYNZ-1983)
  - **Projection Type:** Lambert Conformal Conic (LCC)
  - **Central Meridian:** 84° 15' W
  - **North Standard Parallel:** 38° 58' N
  - **South Standard Parallel:** 37° 58' N
  - **Base Parallel:** 37° 30' N
  - **False Northing:** 0 meters
  - **False Easting:** 500,000 meters
  - **Included Counties:** Anderson, Bath, Boone, Bourbon, Boyd, Bracken, Bullitt, Campbell, Carroll, Carter, Clark, Elliott, Fayette, Fleming, Franklin, Gallatin, Grant, Greenup, Harrison, Henry, Jefferson, Jessamine, Kenton, Lawrence, Lewis, Mason, Menifee, Montgomery, Morgan, Nicholas, Oldham, Owen, Pendleton, Robertson, Rowan, Scott, Shelby, Spencer, Trimble, Woodford
  
- **Zone 2: NAD 83 Kentucky South Zone**
  - **Zone Name:** NAD 83 Kentucky South Zone (KYSZ-1983)
  - **Projection Type:** Lambert Conformal Conic (LCC)
  - **Central Meridian:** 85° 45' W
  - **North Standard Parallel:** 37° 56' N
  - **South Standard Parallel:** 36° 44' N
  - **Base Parallel:** 36° 20' N
  - **False Northing:** 500,000 meters
  - **False Easting:** 500,000 meters
  - **Included Counties:** Adair, Allen, Ballard, Barren, Bell, Boyle, Breathitt, Breckinridge, Butler, Caldwell, Calloway, Carlisle, Casey, Christian, Clay, Clinton, Crittenden, Cumberland, Daviess, Edmonson, Estill, Floyd, Fulton, Garrard, Graves, Grayson, Green, Hancock, Hardin, Harlan, Hart, Henderson, Hickman, Hopkins, Jackson, Johnson, Knott, Knox, Larue, Laurel, Lee, Leslie, Letcher, Lincoln, Livingston, Logan, Lyon, Madison, Magoffin, Marion, Marshall, Martin, McCracken, McCreary, McLean, Meade, Mercer, Metcalfe, Monroe, Muhlenberg, Nelson, Ohio, Owsley, Perry, Pike, Powell, Pulaski, Rockcastle, Russell, Simpson, Taylor, Todd, Trigg, Union, Warren, Washington, Wayne, Webster, Whitley, Wolfe

### C.3 - Series 3: North American Terrestrial Reference Frame of 2022 (NATRF2022)

**Important Note:** *The following series of layers and zones has been proposed to and accepted by NGS as Kentucky's contribution to SPCS2022, but will not become adopted through KRS 1:020 and 200 KAR 041:010 until the terrestrial reference frames defining SPCS2022 have been officially adopted and are supported by NGS. The status of this series is therefore pending.*

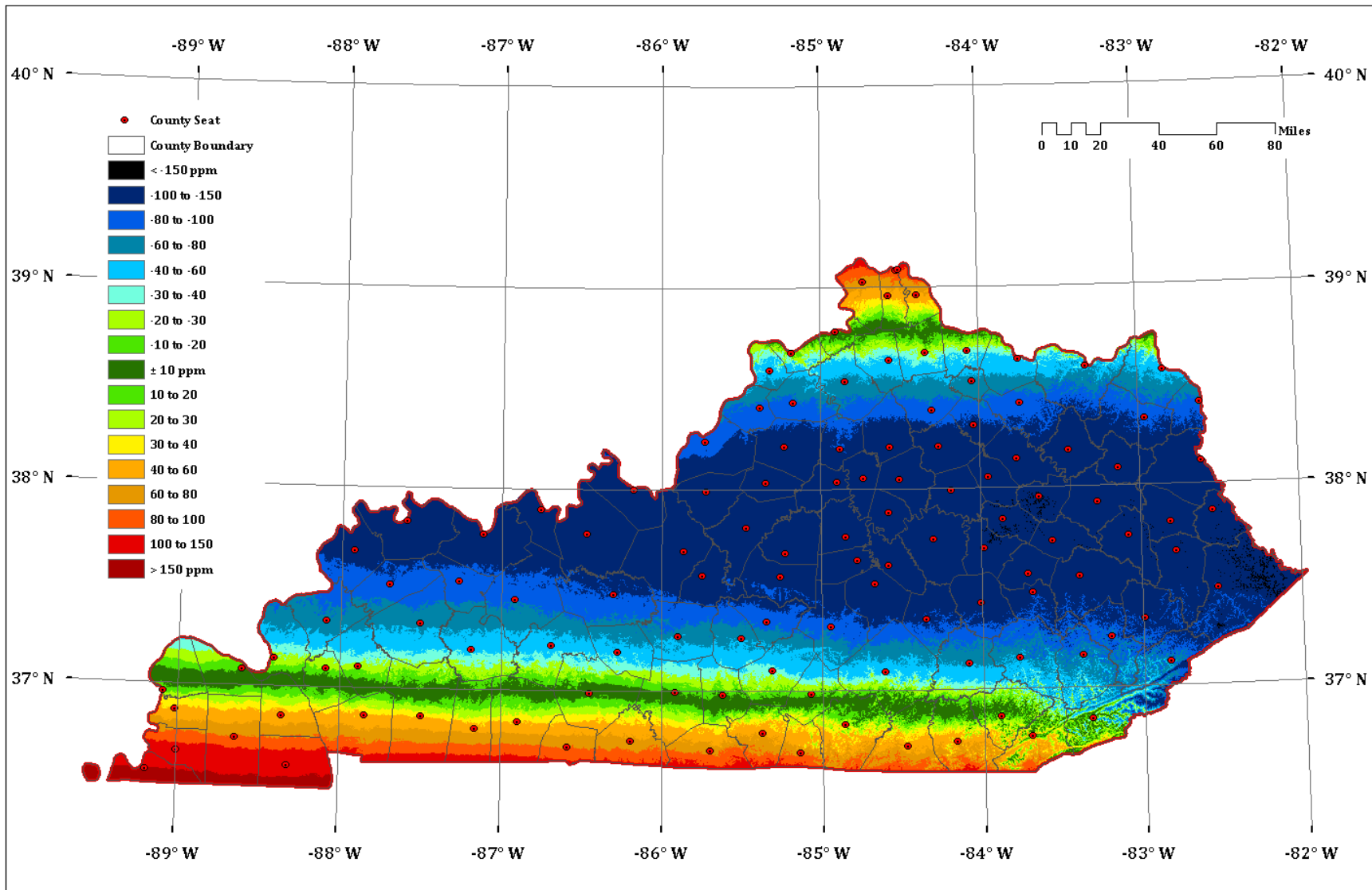
- **Series Name:** Kentucky State Plane Coordinate System of 2022
- **Datum:** North American Terrestrial Frame of 2022 (NATRF2022)
- **Ellipsoid Parameters:**
  - Ellipsoid Name: Geodetic Reference System of 1980 (GRS 80)
  - Radius of Semi-major (equatorial) axis ( $a$ ): 6,378,137 meters (exact).
  - Geometric Inverse of Flattening ( $1/f$ ): 298.257222101 (derived).
- **Linear Units of Measure:**
  - **Defining Unit:** meter.
  - **Unit for Describing Coordinates:** International foot (24 FR 5348, Jun. 30, 1959 and KRS 1:020)
  - **Forward Metric Conversion Factor:** 1 foot = 0.3048 meter
  - **Reverse Metric Conversion Factor:** 1 meter = 3.280839895 feet (computed to 9 decimal places).
- **Layer 1 – NATRF 2022 Kentucky Single Zone**
  - **Layer Name:** Kentucky Single Zone Coordinate System of 2022
  - **Purpose of Layer:** Single zone statewide coverage designed to minimize projection distortions relative to the topographic surface.
  - **Zone 1: NATRF 2022 Kentucky Single Zone**
    - **Zone Name:** NATRF 2022 Kentucky Single Zone (KY1Z-2022)
    - **Projection Type:** Lambert Conformal Conic (LCC)
    - **Central Scale Factor ( $k_0$ ):** 0.999930 (-70 ppm)
    - **Central Meridian:** 85° 45' W
    - **Central Parallel:** 37° 48' N
    - **False Northing:** 1,250,00 meters
    - **False Easting:** 500,000 meters
    - **Included Counties:** Statewide coverage (all counties).
- **Layer 2 – NATRF 2022 Kentucky Low Distortion Projection (LDP) Zones**
  - **Layer Name:** NATRF 2022 Kentucky Low Distortion Projection (LDP) Zones
  - **Purpose of Layer:** Multi-zone statewide coverage designed to achieve low distortion performance relative to the topographic surface.
  - **Zone 1: NATRF 2022 Kentucky LDP Zone 1 – South West**
    - **Zone Name:** NATRF 2022 Kentucky LDP Zone 1 - South West (KYLDP1SW-2022)

- **Projection Type:** Lambert Conformal Conic (LCC)
- **Central Scale Factor ( $k_0$ ):** 1.000000 (0 ppm)
- **Central Meridian:** 271° 45' E (271.15° E)
- **Central Parallel:** 37° 00' N (37.00° N)
- **False Northing:** 125,000 meters
- **False Easting:** 1,200,000 meters
- **Included Counties:** Ballard, Caldwell, Calloway, Carlisle, Christian, Crittenden, Fulton, Graves, Hickman, Hopkins, Livingston, Lyon, McCracken, Marshall, Muhlenberg, Todd, Trigg
  
- **Zone2 : NATRF 2022 Kentucky LDP Zone 2 – South Central**
  - **Zone Name:** NATRF 2022 Kentucky LDP Zone 2 – South Central (KYLDP2SC-2022)
  - **Projection Type:** Lambert Conformal Conic (LCC)
  - **Central Scale Factor ( $k_0$ ):** 1.000025 (25 ppm)
  - **Central Meridian:** 274° 21' E (274.35° E)
  - **Central Parallel:** 37° 03' N (37.05° N)
  - **False Northing:** 125,000 meters
  - **False Easting:** 1,520,000 meters
  - **Included Counties:** Adair, Allen, Barren, Butler, Casey, Clinton, Cumberland, Edmonson, Green, Hart, Logan, McCreary, Metcalfe, Monroe, Pulaski, Russell, Simpson, Taylor, Warren, Wayne
  
- **Zone 3: NATRF 2022 Kentucky LDP Zone 3 – South East**
  - **Zone Name:** NATRF 2022 Kentucky LDP Zone 3 – South East (KYLDP3SE-2022)
  - **Projection Type:** Oblique Mercator (OM)
  - **Central Scale Factor ( $k_0$ ):** 1.000035 (35 ppm)
  - **Central Meridian:** 276° 45' E (276.75° E)
  - **Central Parallel:** 37° 27' N (37.45° N)
  - **Projection Axis Skew Azimuth:** 60°
  - **False Northing:** 150,000 meters
  - **False Easting:** 1,810,000 meters
  - **Included Counties:** Bell, Breathitt, Clay, Floyd, Harlan, Jackson, Johnson, Knott, Knox, Laurel, Lee, Leslie, Letcher, Magoffin, Martin, Owsley, Perry, Pike, Rockcastle, Whitley, Wolfe
  
- **Zone 4: NATRF 2022 Kentucky LDP Zone 4 – Mid West**
  - **Zone Name:** NATRF 2022 Kentucky LDP Zone 4 – Mid West (KYLDP4MW-2022)
  - **Projection Type:** Lambert Conformal Conic (LCC)
  - **Central Scale Factor ( $k_0$ ):** 1.000015 (15 ppm)
  - **Central Meridian:** 273° 24' E (273.40° E)
  - **Central Parallel:** 37° 42' N (37.70° N)
  - **False Northing:** 375,000 meters
  - **False Easting:** 1,200,000 meters
  - **Included Counties:** Breckinridge, Daviess, Grayson, Hancock, Hardin, Henderson, Larue, McLean, Marion, Meade, Nelson, Ohio, Union, Washington, Webster

- **Zone 5: NATRF 2022 Kentucky LDP Zone 5 – Mid Central**
  - **Zone Name:** NATRF 2022 Kentucky LDP Zone 5 – Mid Central (KYLDP5MC-2022)
  - **Projection Type:** Oblique Mercator (OM)
  - **Central Scale Factor ( $k_0$ ):** 1.000025 (25 ppm)
  - **Central Meridian:** 275° 36' E (275.60° E)
  - **Central Parallel:** 38° 00' N (38.00° N)
  - **Projection Axis Skew Azimuth:** 70°
  - **False Northing:** 375,000 meters
  - **False Easting:** 1,520,000 meters
  - **Included Counties:** Anderson, Bourbon, Boyle, Clark, Estill, Fayette, Franklin, Garrard, Harrison, Jessamine, Lincoln, Madison, Mercer, Nicholas, Powell, Scott, Woodford
  
- **Zone 6: NATRF 2022 Kentucky LDP Zone 6 – North East**
  - **Zone Name:** NATRF 2022 Kentucky LDP Zone 6 – North East (KYLDP6NE-2022)
  - **Projection Type:** Lambert Conformal Conic (LCC)
  - **Central Scale Factor ( $k_0$ ):** 1.000025 (25 ppm)
  - **Central Meridian:** 276° 39' E (276.65° E)
  - **Central Parallel:** 38° 21' N (38.35° N)
  - **False Northing:** 375,000 meters
  - **False Easting:** 1,810,000 meters
  - **Included Counties:** Bath, Boyd, Bracken, Carter, Elliott, Fleming, Greenup, Lawrence, Lewis, Mason, Menifee, Montgomery, Morgan, Robertson, Rowan
  
- **Zone 7: NATRF 2022 Kentucky LDP Zone 7 – North Central**
  - **Zone Name:** NATRF 2022 Kentucky LDP Zone 7 – North Central (KYLDP7NC-2022)
  - **Projection Type:** Oblique Mercator (OM)
  - **Central Scale Factor ( $k_0$ ):** 1.000000 (0 ppm)
  - **Central Meridian:** 274° 57' E (274.95° E)
  - **Central Parallel:** 38° 30' N (38.50° N)
  - **Projection Axis Skew Azimuth:** 50°
  - **False Northing:** 625,000 meters
  - **False Easting:** 1,520,000 meters
  - **Included Counties:** Boone, Bullitt, Campbell, Carroll, Gallatin, Grant, Henry, Jefferson, Kenton, Oldham, Owen, Pendleton, Shelby, Spencer, Trimble

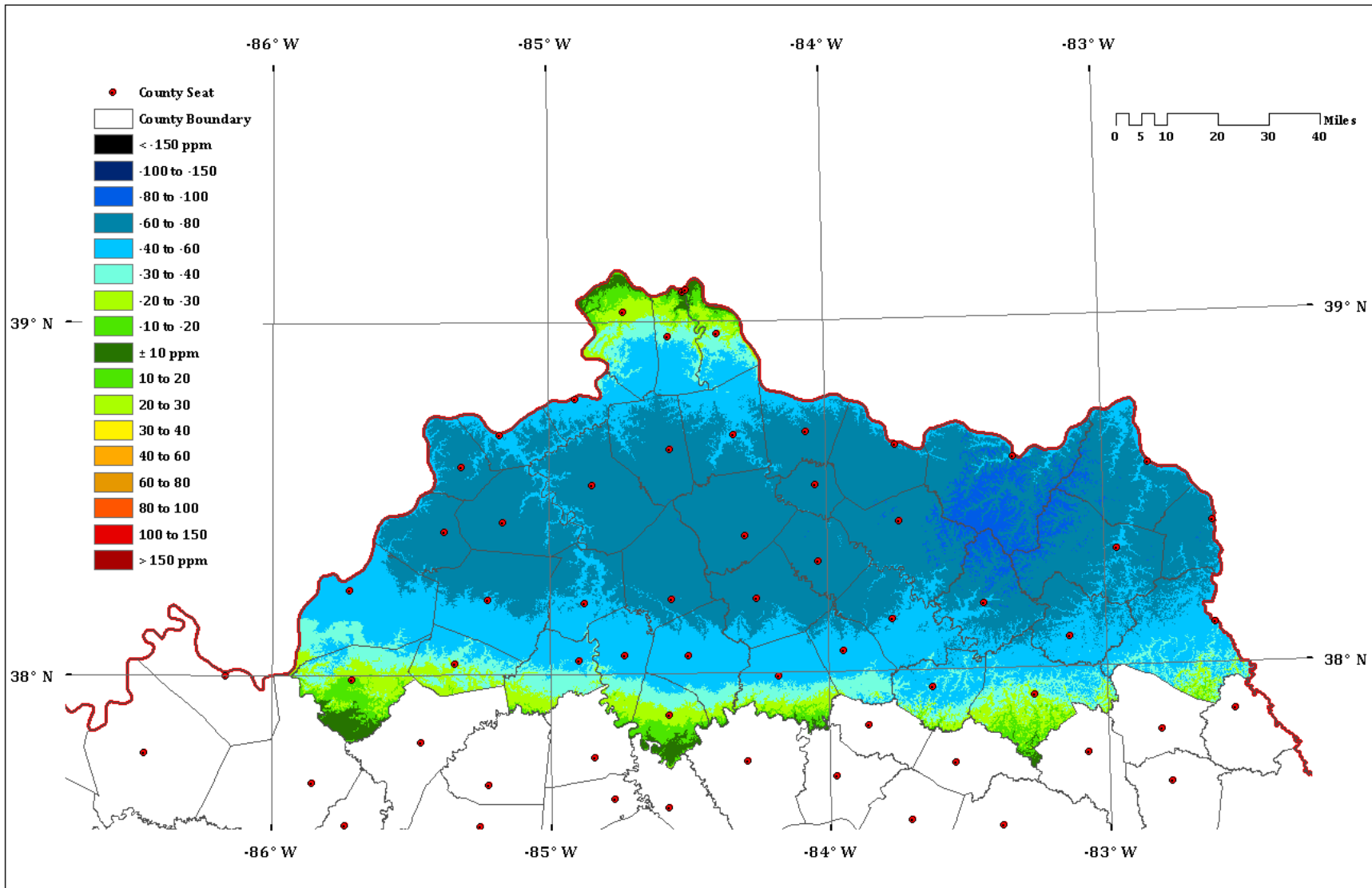
## **Appendix D**

### **D.1 - Projection Distortion Performance Maps**

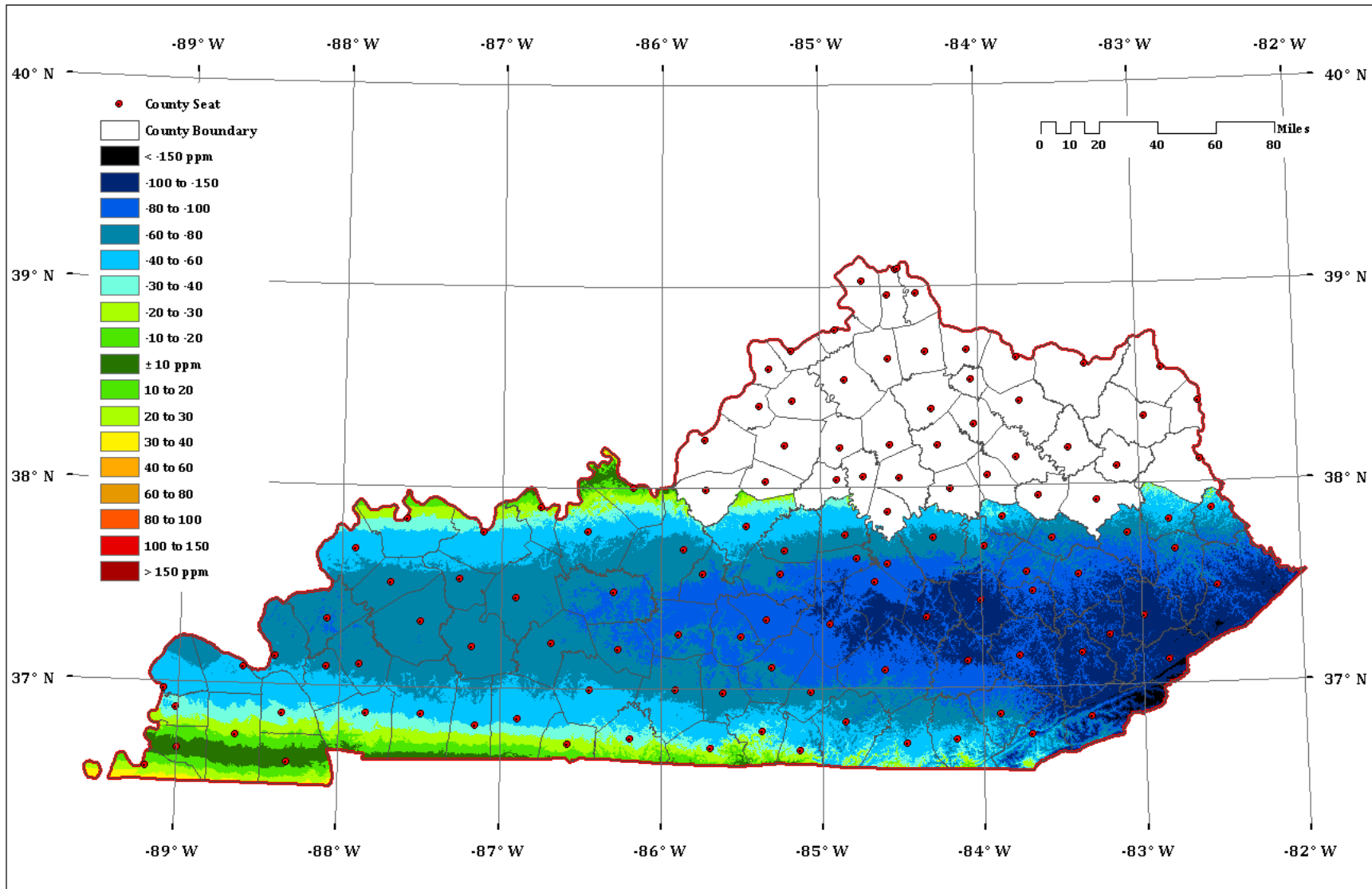


NAD 1983 Kentucky Single Zone (KY1Z-1983)

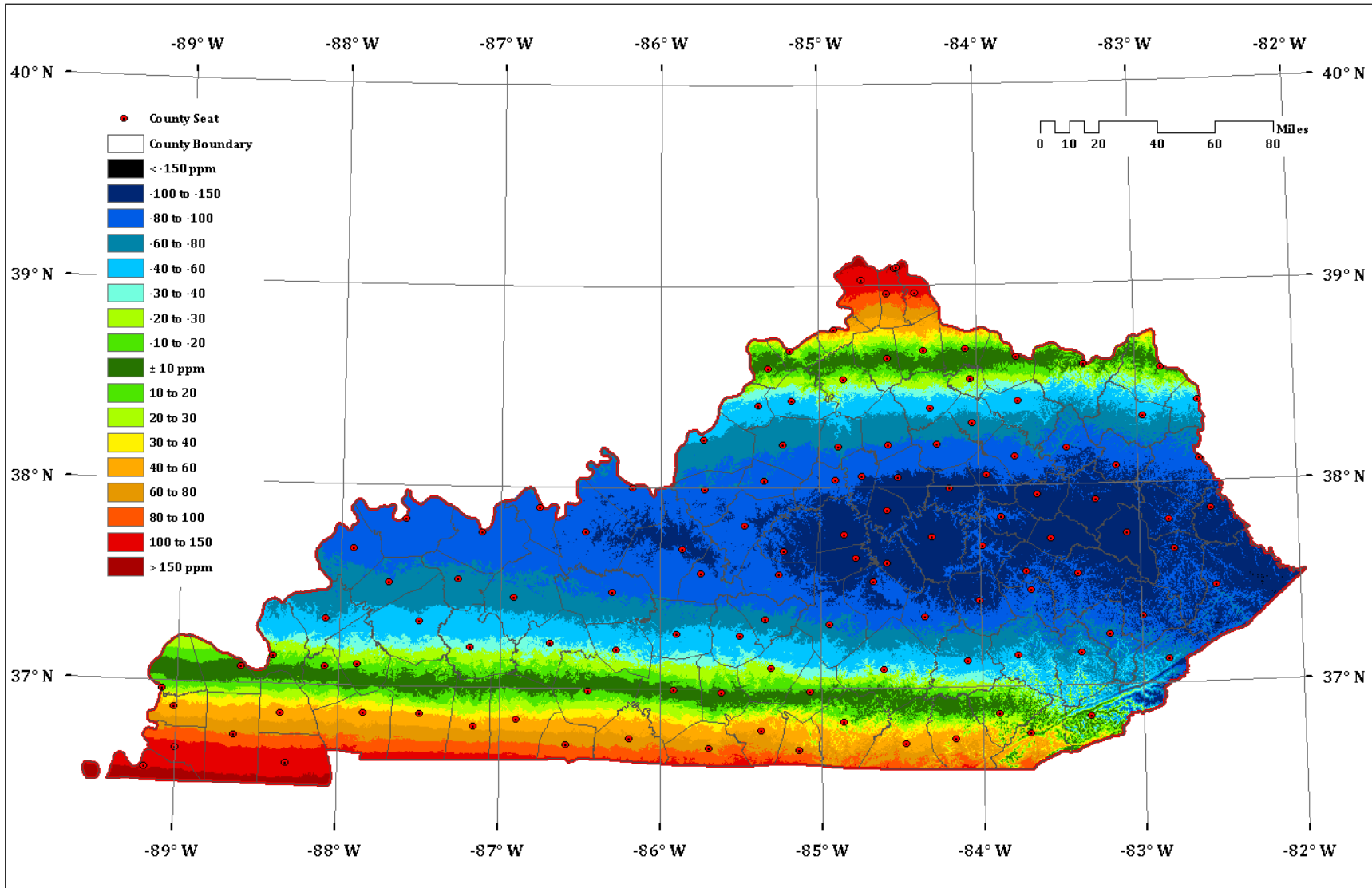




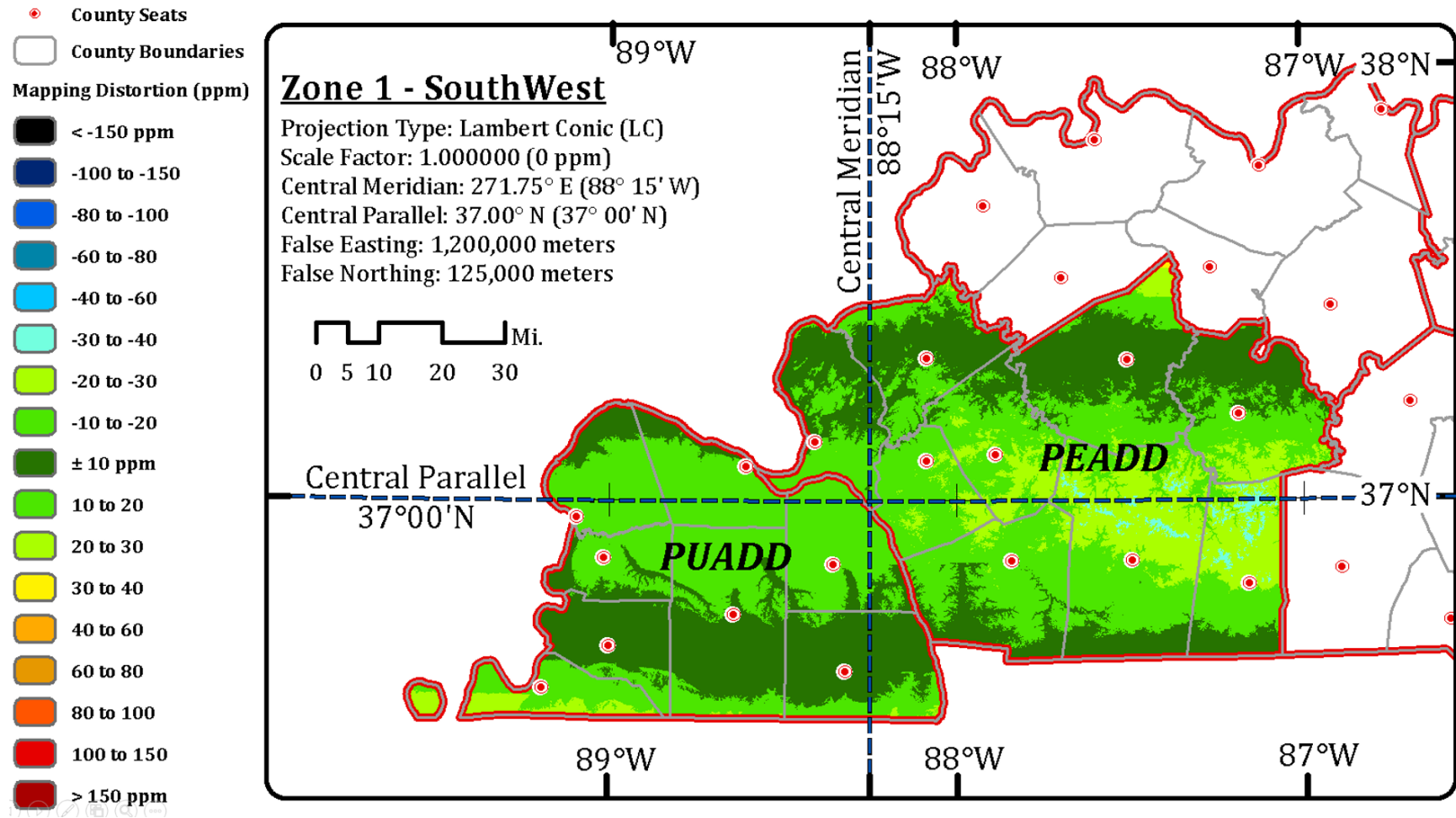
NAD 1983 Kentucky North Zone (KYNZ-1983)



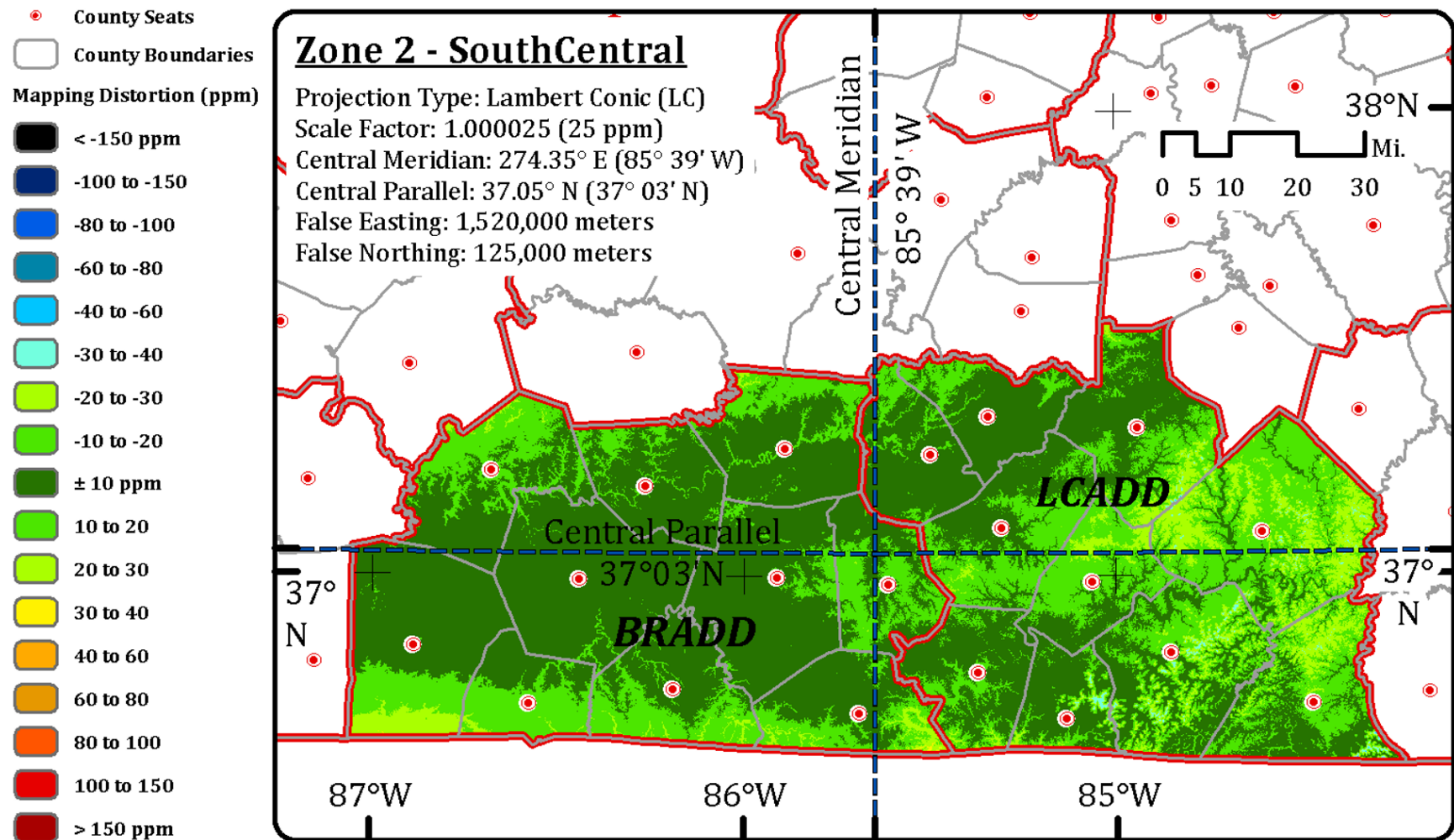
NAD 1983 Kentucky South Zone (KYSZ-1983)



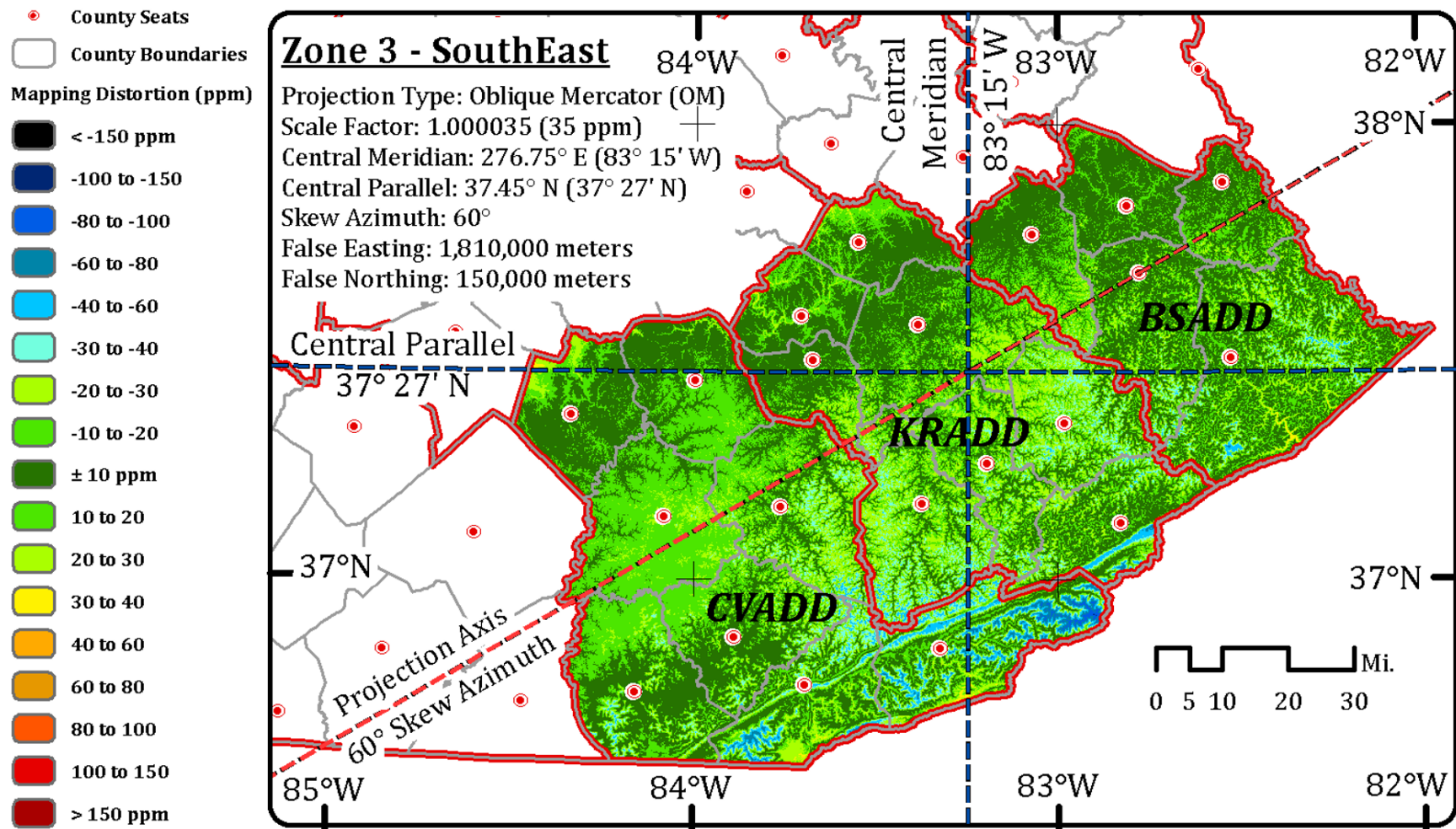
**NATRF 2022 Kentucky Single Zone (KY1Z-2022; NGS: 210001 KY)**



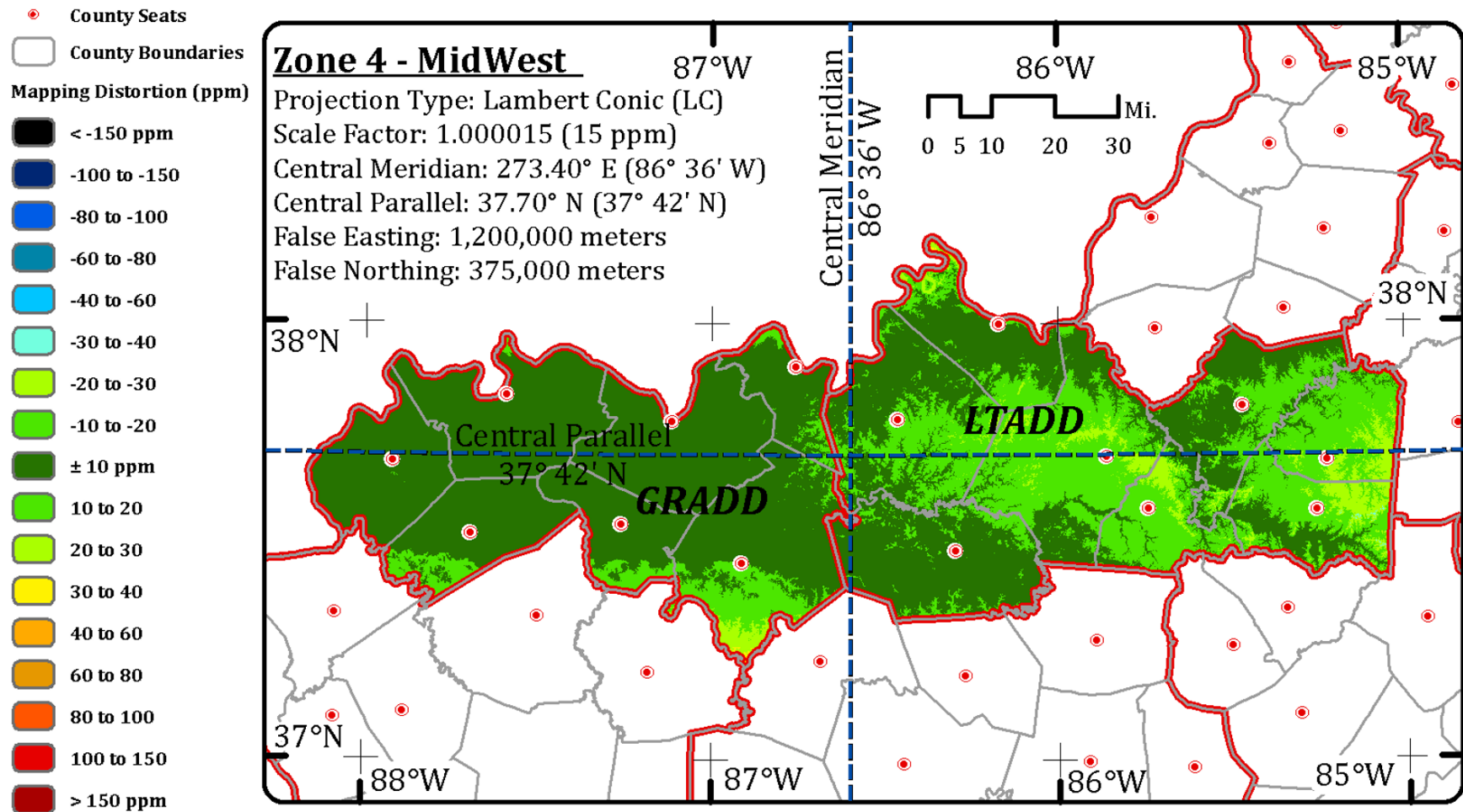
NATRF 2022 Kentucky LDP Zone 1 - South West (KYLDP1SW-2022; NGS: 211001 KY SW)



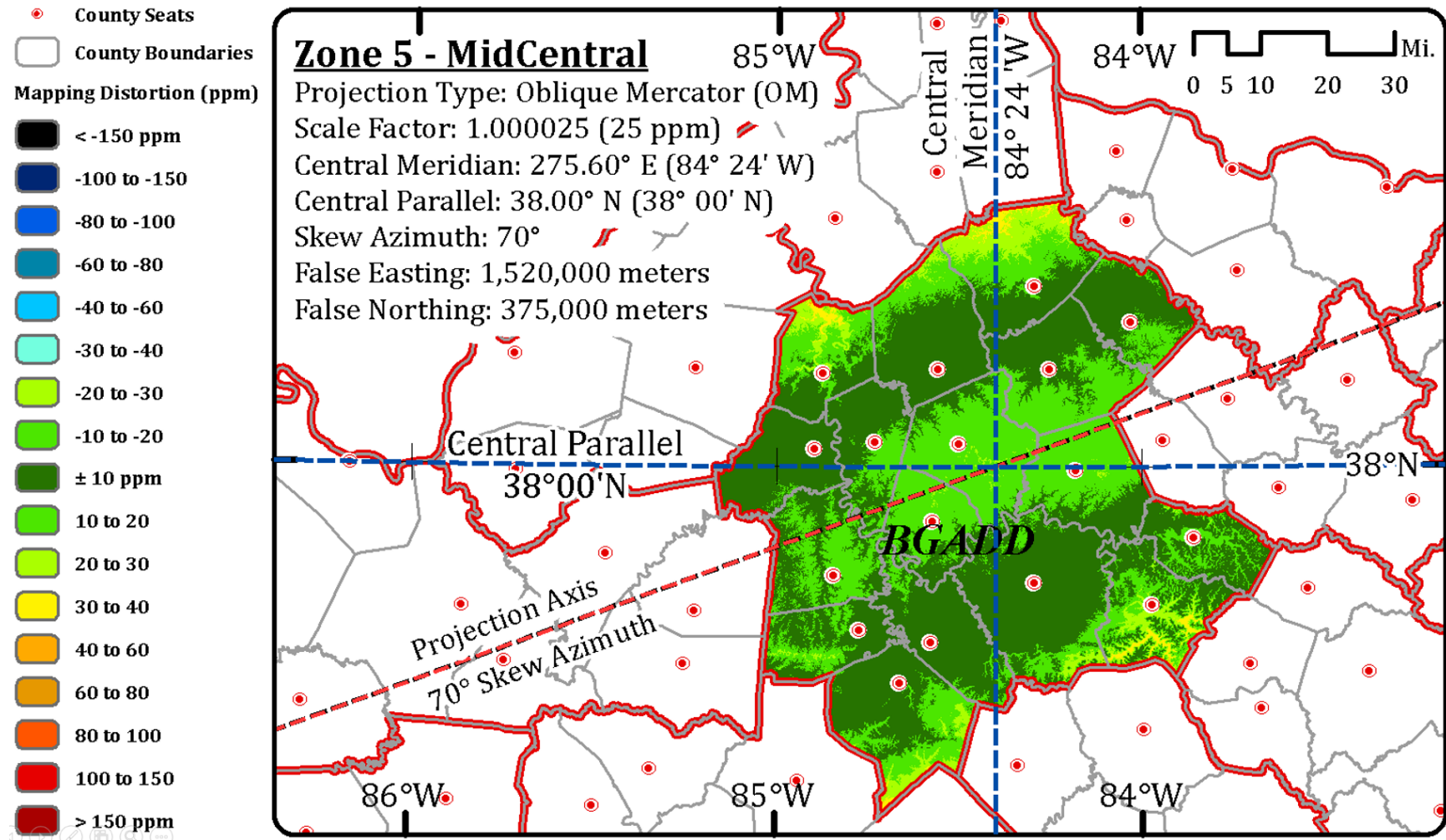
NATRF 2022 Kentucky LDP Zone 2 - South Central (KYLDP2SC-2022; NGS: 211002 KY SC)



NATRF 2022 Kentucky LDP Zone 3 - South East (KYLDP3SE-2022; NGS: 211003 KY SE)

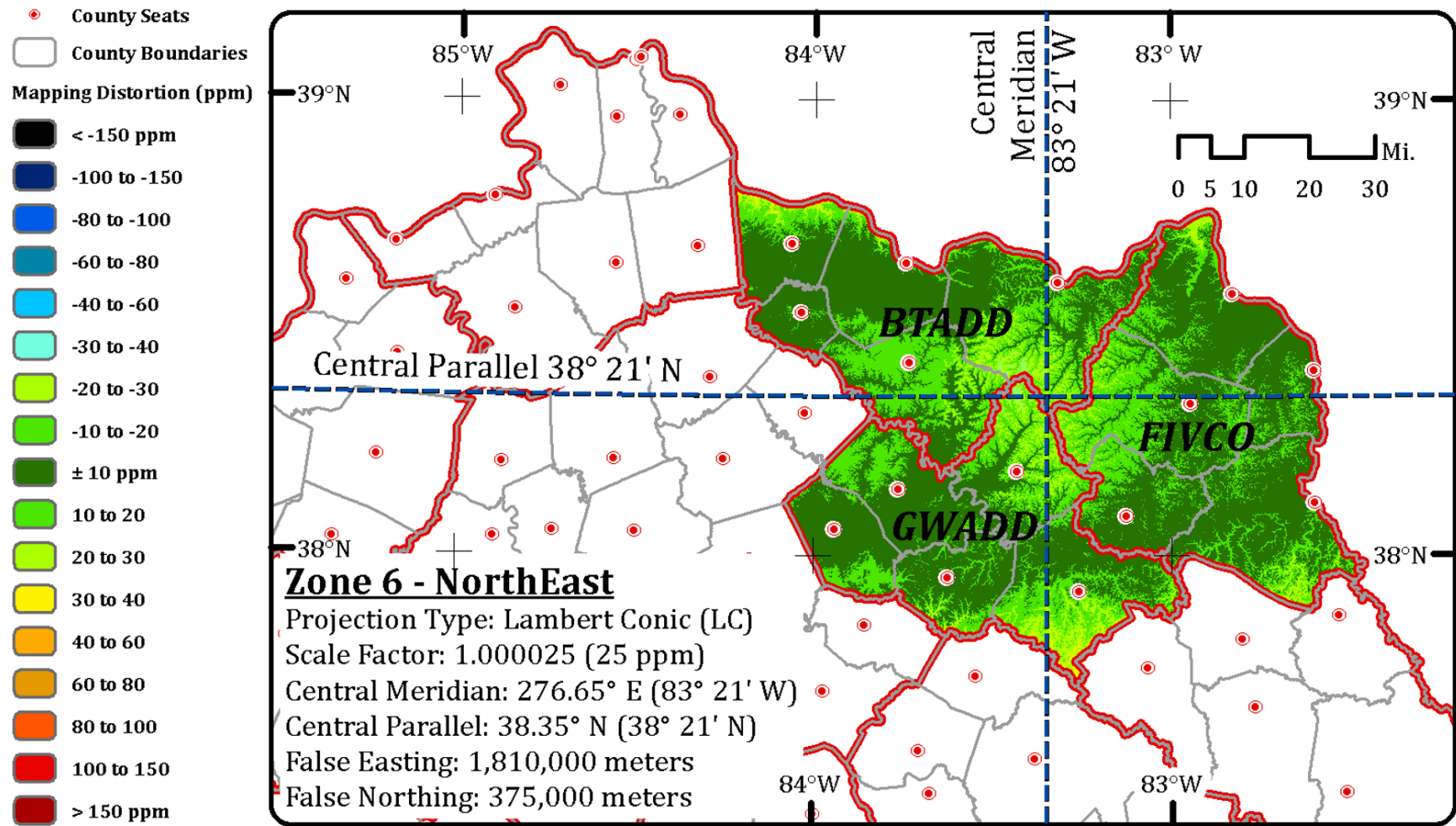


NATRF 2022 Kentucky LDP Zone 4 - Mid West (KYLDP4MW-2022; NGS: 211004 KY MW)

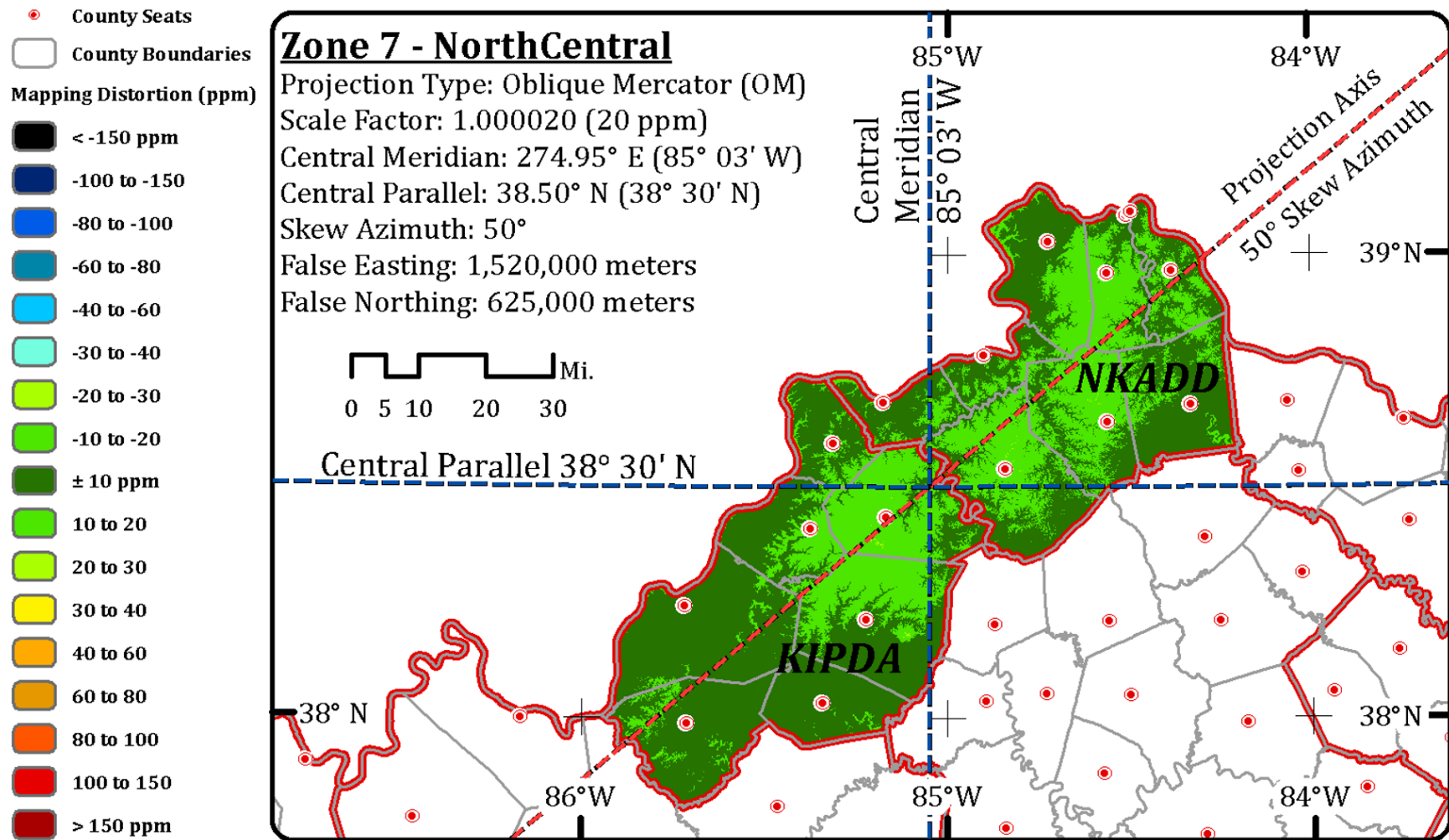


NATRF 2022 Kentucky LDP Zone 5 - Mid Central (KYLDP5MC-2022, NGS 211005 KY MC)





NATRF 2022 Kentucky LDP Zone 6 - North East (KYLDP6NE-2022; NGS: 211006 KY NE)



NATRF 2022 Kentucky LDP Zone 7 - North Central (KYLDP7NC-2022; NGS: 211007 KY NC)

# Appendix E: Six Steps Illustrating Low Distortion Projection (LDP) Design

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The design objective is usually to minimize linear distortion over the largest area possible. These goals are at odds with one another, so LDP design is an **optimization problem**. It is important to also realize that the most difficult part is often not technical but psychological. There is little value in designing an LDP for a region without first getting concurrence and buy-in from the many stakeholders impacted by the design. This includes surveyors, engineers, GIS professionals, as well as public and private organizations that make use of geospatial data in the design area. Getting stakeholders involved early in the process will increase the likelihood that the LDP will be adopted and actually used.

The following six steps are intended to illustrate commonly encountered situations in LDP design. These steps are provided to teach the design concepts; in the actual design process some of these “steps” can be omitted or modified, especially when designing for large areas. But these steps work well for small areas (< ~30 miles or 50 km wide perpendicular to the projection axis).

## E.1 - Determine distortion objective and representative ellipsoid height for area of interest

**NOTE:** *This is just to get the design process started.* Ellipsoid height by itself is unlikely to yield the final design scale, except for small areas, due to curvature and/or systematic change in topographic height. It is even possible to skip this step entirely, and instead start the process with a projection scale of 1 (or some other arbitrary value). However, considering height helps illustrate the concepts behind design the process.

- A common objective for “low distortion” is  $\pm 20$  ppm ( $\pm 0.1$  ft/mile), but this may not be achievable due to range of topographic height and/or size of design area. The following “rules of thumb” can help guide the initial design. However, it may be possible to achieve better results than these guidelines indicate, because both height and areal extent affect distortion simultaneously, and one can be used to compensate for the other.
- Size of design area. **Distortion due to curvature is within  $\pm 5$  ppm for an area 35 miles wide.** Note that this width is perpendicular to the projection axis (e.g., east-west for TM and north-south for LCC projections). The effect is not linear; range of distortion due to curvature increases rapidly with increasing zone width and is proportional to the square of the zone width, i.e., doubling the zone width increases the distortion by about a factor of four (for this  $\pm 5$  ppm case, doubling zone width to 70 miles quadruples the distortion range to about  $\pm 20$  ppm).

- Range in topographic ellipsoid height. **Distortion due to change in topographic height is about  $\pm 5$  ppm for a  $\pm 100$  ft range in height.** Note that this is linear for the topographic height ranges on Earth. Thus, a range of  $\pm 400$  ft in height corresponds to a range of about  $\pm 20$  ppm distortion.
- The *average* height of an area may not be appropriate (e.g., because of mountains in the design area).
  - There is usually no need to estimate height to an accuracy of better than about  $\pm 20$  ft ( $\pm 6$  m); this corresponds to about  $\pm 1$  ppm distortion. In addition, the initial projection scale determined using this height will likely be refined later in the design process.

## E.2 - Choose projection type and place projection axis near centroid of project area

**NOTE:** *This is just to get the design process started.* In cases where the topography generally changes in one direction, offsetting the projection axis can yield substantially better results. As with step #1, there is no need to spend a lot of effort on this step, since the effect of the projection type and axis location is evaluated later in the design process.

- Select a well-known and widely used **conformal** projection, such as the Transverse Mercator (TM), Lambert Conformal Conic (LCC), or Oblique Mercator (OM).
  - When minimizing distortion, it is not always obvious which projection type to use, but for small areas ( $< \sim 30$  miles or  $\sim 50$  km wide), both the TM and LCC will usually provide similar and satisfactory results. However, significantly better performance can be obtained in many cases when a projection is used with its axis perpendicular to the general topographic slope of the design area (more on this below).
  - In nearly all cases, a two-parallel LCC should **not** be used for an LDP (but note that some software may not support a one-parallel LCC). A two-parallel LCC should not be used because the reason there are two parallels is to make the projection secant to the ellipsoid (i.e., to make the central parallel scale less than 1). This is at odds with the usual objective of scaling the projection so that the developable surface is at the topographic surface, which is typically well above the ellipsoid, particularly in areas where reduction in distortion is desired. Even for LDP designs that use secant LCC definitions, it is easier to design an LDP using a one-parallel rather than two-parallel LCC.
  - The OM projection can be very useful for minimizing distortion over large areas, especially areas that are elongate in an oblique direction. It can also be useful in areas where the topographic slope varies gradually and more-or-less uniformly in an oblique direction. The disadvantage of this projection is that it is more difficult to use for designs that account for topographic slope, since the projection skew axis location, orientation, and scale must be simultaneously optimized. Such designs would be extremely difficult to perform manually but can be optimized using mathematical methods (such as least squares). There is also more than one version of the OM projection; the Hotine OM, also called Rectified Skew Orthomorphic (RSO),

- is the most common version of the OM used in the U.S. (and it is the type used for State Plane).
- The oblique stereographic projection can also be used, but it is unlikely that it will perform better than the TM, LCC, or OM projections since it does not curve with the Earth in any direction. Situations where it would provide the lower distortion than the other projections would only rarely (if ever) be encountered. In addition, there are two common versions (“original” and “double” stereographic), but they do not yield the same coordinates and so care must be taken to ensure the one used for design is the same used in subsequent applications (coordinates differ by about 1 foot at locations 20 miles from the projection origin).
  - When choosing a projection, universal commercial software support, although desirable, is not an essential requirement. In rare cases where third parties must use a coordinate system based on a projection not supported in their software, it is possible for them to get on the coordinate system implicitly, for example by using a 2D best-fit conformal transformation based on LDP coordinates at common points (e.g., the so-called horizontal “calibration” or “localization” process available in most commercial GNSS surveying software).
  - Placing the projection axis near the design area centroid is often a good first step in the design process (or, for the OM projection, parallel to the long axis of the design area).
    - In cases where topographic height increases more-or-less uniformly in one direction, dramatically better performance can be achieved by offsetting the projection axis from the project centroid. In such cases a projection type should be chosen such that its projection axis is perpendicular to the topographic slope (e.g., for topography sloping east-west, a TM projection should be used; for slope north-south, an LCC projection should be used). The axis is located such that the developable surface best coincides with the topographic surface (as shown in Figure 4 for an LCC).
    - Often the central meridian of the projection is placed near the east-west “middle” of the project area in order to minimize convergence angles (i.e., the difference between geodetic and grid north). The central meridian is the projection axis only for the TM projection; its location has no effect on linear distortion for the LCC projection.

### E.3 - Scale projection axis to the representative ground height, $h_0$

**NOTE:** *This is just to get the design process started.* Ellipsoid height by itself is unlikely to yield the final design scale, except for small areas, due to curvature and/or systematic change in topographic height. This step can also be skipped by simply starting with  $k_0 = 1$ , but the following provides the concepts (as well as some mathematical information for step #4).

- Compute map projection axis scale factor “at ground”:  $k_0 = 1 + \frac{h_0}{R_G}$ 
  - For TM projection,  $k_0$  is the central meridian scale factor.
  - For one-parallel LCC projection,  $k_0$  is the standard (central) parallel scale factor.

- For OM projection,  $k_0$  is the scale at the local origin.
- $R_G$  is the geometric mean radius of curvature,  $R_G = \frac{a\sqrt{1-e^2}}{1-e^2 \sin^2 \varphi}$

where  $\varphi$  = geodetic latitude of point, and for the GRS 80 ellipsoid:

$$a = \text{semi-major axis} = 6,378,137 \text{ m (exact)} = 20,925,646.325 \text{ international ft} \\ = 20,925,604.474 \text{ U.S. survey ft}$$

$$e^2 = \text{first eccentricity squared} = f(2 - f) = 0.00669438002290\dots$$

$$f = \text{geometric flattening} = 1 / 298.257222101$$

- Alternatively, can initially approximate  $R_G$  using Table 5, since  $k_0$  will likely be refined in Step #4:

**Table 5.** Geometric mean radius of curvature at various latitudes for the GRS 80 ellipsoid (rounded to nearest 1000 feet and meters).

Lat	feet	(meters)	Lat	feet	(meters)	Lat	feet	(meters)
0°	20,855,000	(6,357,000)	35°	20,902,000	(6,371,000)	65°	20,971,000	(6,392,000)
10°	20,860,000	(6,358,000)	40°	20,913,000	(6,374,000)	70°	20,980,000	(6,395,000)
15°	20,865,000	(6,360,000)	45°	20,926,000	(6,378,000)	75°	20,987,000	(6,397,000)
20°	20,872,000	(6,362,000)	50°	20,938,000	(6,382,000)	80°	20,992,000	(6,398,000)
25°	20,880,000	(6,364,000)	55°	20,950,000	(6,385,000)	85°	20,995,000	(6,399,000)
30°	20,890,000	(6,367,000)	60°	20,961,000	(6,389,000)	90°	20,996,000	(6,400,000)

#### E.4 - Compute distortion throughout project area and refine design parameters

- Distortion computed at a point (at ellipsoid height  $h$ ) as  $\delta = k \left( \frac{R_G}{R_G + h} \right) - 1$ 
  - Where  $k$  = projection grid point scale factor (i.e., distortion with respect to the ellipsoid at a point). Note that computation of  $k$  is rather involved, and is often done in commercial software. However, if your software does not compute  $k$ , or if you want to check the accuracy of  $k$  computed by your software, equations for doing so are included in the mapping equations for LCC, TM, and OM projections in Appendix F.
  - Multiply  $\delta$  by 1,000,000 to get distortion in parts per million (ppm).
- Best approach is to compute distortion over entire area and generate a distortion map and compute distortion statistics (this helps ensure low-distortion coverage is achieved where it is desired).
- Often requires repeated evaluation using different  $k_0$  values and different projection axis locations.

- May also warrant trying different projection types.
- General approach for computational refinement:
  - Compute distortion statistics, such as mean, range, and standard deviation for all points in the design area.
  - Changing the projection scale only affects the mean distortion; it has essentially no effect on the variability (standard deviation and range).
  - The only way to reduce distortion variability is by moving the projection axis and/or changing the projection type. The usual objective is to minimize the distortion range. Once this is done, the scale can be changed so that the mean distortion is near zero (this will have no effect on the distortion range or standard deviation).
  - Finally, check to ensure the desired distortion is achieved in important areas, and check to ensure overall performance is satisfactory, by using a map showing distortion everywhere in the design area. It may be worthwhile to give greater weight to distortion in populated areas (such as cities), rather than using the same weight for all areas.

### **E.5 - Keep the definition SIMPLE and CLEAN!**

- Define  $k_0$  to no more than SIX decimal places, e.g., 1.000175 (exact).
  - *Note:* A change of one unit in the sixth decimal place ( $\pm 1$  ppm) equals distortion caused by a 20 ft (6 m) change in height.
  - For large areas with variable relief, scale defined to five decimal places ( $\pm 10$  ppm) is often sufficient.
- Define the central meridian and latitude of grid origin to nearest whole arc-minute. Using arc-minutes evenly divisible by 3 will result in exact values in decimal degrees (e.g.,  $121^\circ 33' 00''$  W =  $-121.55^\circ$ ), although some prefer using the nearest 5 arc-minutes (as done for State Plane 1983 and 1927).
- Define grid origin using whole values. Often it is desired to use values with as few digits as possible (e.g., false easting = 50,000 for a system with maximum easting coordinate value < 100,000), although there are many different options for selecting values. Note that the grid origin definition has no effect whatsoever on map projection distortion.
  - It is strongly recommended that the coordinate values everywhere in the design area be distinct from other coordinate system values for that area (such as State Plane and UTM) in order to reduce the risk of confusing the LDP with other systems. For multi-zone LDPs, it could similarly be helpful to keep coordinates between the zones distinct, if possible.
  - It may be desirable to define grid origins such that the northings and eastings do not equal one another anywhere in the design area.
  - In some applications, there may be an advantage to using other criteria for defining the grid origin. For example, it may be desirable for all coordinates in the design area

to have the same number of digits (such as six digits, i.e., between 100,000 and 999,999). In other cases, it may be useful to make the coordinates distinct from State Plane by using larger rather than smaller coordinates, especially if the LDP covers a very large area. In multi-zone systems, it may also be helpful to define grid origins such that the values correlate to zone numbers (e.g., coordinates between 3,000,000 3,999,999 m for a zone designated as #3). This approach was used for the Kansas Regional Coordinate System (Dennis, 2017).

## E.6 - Explicitly define linear unit and geometric reference system (i.e., geodetic datum)

- Linear unit, e.g., meter (or international foot, or US survey foot, or...?)
  - Although the U.S. survey foot is currently used for SPCS 83 in most states, that linear unit will be officially deprecated by the U.S. government on December 31, 2022. That means the U.S. survey foot cannot be used for projection definitions that will become part of the State Plane Coordinate System of 2022, and it will not be supported by NGS for any component of the National Spatial Reference System (NSRS) after 2022 (including elevations). However, the U.S. survey foot will continue to be supported as a legacy unit by NGS in applications that compute State Plane coordinates for zones where it was officially specified for SPCS 83, and for all zones of SPCS 27.
  - The foot definition used after 2022 will be simply be called the “foot”, and it will be numerically identical to the foot definition presently called the “international foot” (i.e., 1 foot = 0.3048 meter *exactly*). The intent is to have a single, uniform definition of the foot used throughout the U.S. for *all* applications. However, in applications where confusion over the type of foot can occur (such as surveying and mapping), continued use of the term “international foot” is recommended.
- Geometric reference system (datum), e.g., North American Datum of 1983 (NAD 83)
  - The reference system realization (“datum tag”) and epoch date (e.g., 2010.00) should not be included in the coordinate system definition (just as it is not included in State Plane definitions). However, the datum tag and epochs *are* essential components for defining the spatial data used within the coordinate system. For NAD 83, the NGS convention is to give the datum tag in parentheses after the datum name, usually as the year in which the datum was “realized” as part of a network adjustment. Epoch dates are given after the datum tag and are preceded by the word “epoch.” Although given as decimal years, they are usually not the same as the datum tag. Common datum tags and epochs for NGS control are listed below. Prior to the NAD 83 (2011) epoch 2010.00 realization, epochs were only used for tectonically active areas and CORS. But they will be used for all components of the NSRS after 2022. Below are some common datum tags and epochs for geometric (“horizontal”) geodetic control:
    - “2011” for the current NAD 83 (2011) epoch 2010.00 realization, which is referenced to the North America tectonic plate. A tag of “PA11” is used for control referenced to the Pacific plate (e.g., Hawaii, American Samoa), and a tag of “MA11” is used for control referenced to the Pacific plate (e.g., Guam).



- “2007” for the (superseded) NSRS2007 (National Spatial Reference System of 2007) realization. Functionally equivalent to the superseded “CORS” datum tag and referenced to an epoch of 2002.00 for most of the coterminous US and the Caribbean (an epoch of 2007.00 was used for the western states of AK, AZ, CA, NV, OR, and WA).
  - “199x” for most of the various superseded HARN (or HPGN) and Federal Base Network (FBN) realizations, where x is the last digit of the year of the adjustment (usually done for a particular state). HARN is “High Accuracy Reference Network” and HPGN is “High Precision Geodetic Network”.
- *Note regarding the State Plane Coordinate System of 2022 (SPCS2022):* NGS will replace NAD 83 with new terrestrial reference frames (TRFs) in 2022. The one for North America will be called the North American Terrestrial Reference Frame of 2022 (NATRF2022); there will also be a TRF for the Caribbean, Pacific, and Mariana tectonic plates. The GRS 80 ellipsoid will continue to be used for SPCS2022. In North America, horizontal coordinates will change by less than 2 m (6.5 ft). Ellipsoid heights will change by less than  $\pm 2$  m everywhere. A change in height of 2 m will change linear distortion by 0.3 ppm. Since the change to the 2022 TRFs will have negligible impact on the distortion of LDPs designed with respect to NAD 83, those LDPs could continue to be used with the 2022 datum. However, to avoid confusion it would be prudent to change the grid coordinates so that LDP coordinates based on the 2022 datum are significantly different from those referenced to NAD 83. Such a change will not affect distortion but would reduce the risk of accidentally referencing the wrong datum.

NGS is currently in the process of defining SPCS2022. The references section of this document includes recently released NGS documents about SPCS2022:

- A report on the history, status, and possible future of State Plane (Dennis, 2018).
- New SPCS2022 policy and procedures (NGS, 2019a and 2019b, respectively), which allow for the use of LDPs for SPCS2022 zones. However, the LDPs must be defined by the states where they will be used (NGS will not design zone with a distortion design criterion of less than  $\pm 50$  ppm, due to lack of resources).
- *Note regarding the relationship between NAD 83 and WGS 84:* For the purposes of entering the LDP projection parameters into vendor software, the datum should be defined as NAD 83 (which uses the GRS 80 reference ellipsoid for all realizations). Some commercial software implementations assume there is no transformation between WGS 84 and NAD 83 (i.e., all transformation parameters are zero). Other implementations use a non-zero transformation, and in some cases both types are available. The type of transformation used will depend on specific circumstances, although often the zero transformation is the appropriate choice (even though it is not technically correct). Check with software technical support to ensure the appropriate transformation is being used for your application. Additional information about WGS 84 is available from the National Geospatial-Intelligence Agency (NGA, 2014).
- *Note regarding the vertical component of a coordinate system definition:* The vertical reference system (datum) is an essential part of a three-dimensional coordinate system definition. But LDPs are restricted exclusively to horizontal coordinates. Although the

vertical component is essential for most applications, it is not part of an LDP and must be defined separately. It should be specified as part of the overall coordinate system metadata (as shown in the metadata example later in this document). A complete three-dimensional coordinate system definition must include a vertical “height” component. Typically, the vertical part consists of ellipsoid heights relative to NAD 83 (when using GNSS) and/or orthometric heights (“elevations”) relative to the North American Vertical Datum of 1988 (NAVD 88). These two types of heights are related (at least in part) by a hybrid geoid model, such as GEOID18, and often a vertical adjustment or transformation is needed to match local vertical control for a project. The approach used for the vertical component usually varies from project to project and requires professional judgment to ensure it is defined correctly. Providing such instructions is beyond the scope of this document.

## Appendix F: SPCS Conformal Mapping Projection Equations

### Important Note:

The equations presented in this appendix have been adapted from *NOAA Manual NOS NGS 5 – State Plane Coordinate System of 1983* by James E. Stem (NGS 1990), with modifications to optimize implementation within common computer programming languages such as C# or VB. Other modifications include adaptation for specifying positive east values for the central meridian, specifying a single central parallel for the Lambert conformal conic projection, and specifying false origin coordinates ( $N_c$ ,  $E_c$ ) for the oblique Mercator projection at the local origin as opposed to the natural origin.

It is not within the scope of this appendix to present the background of or derivations for these equations, but to present them exactly as utilized in design of KSPCS2022 and analyzing the resulting projection performance. To better understand the origin of these equations the reader is directed to the above referenced document specifically as they pertain to the SPCS, and for a general understanding of these and other relevant mapping projections the seminal work by John P. Snyder titled *Map Projections – A Working Manual, U.S. Geological Survey Professional Paper 1395* (USGS, 1987) is highly recommended.

### Ellipsoid Specific Constants (used for all projection types in Appendix F)

Note: numerical values given are for the GRS 80 geodetic reference ellipsoid (used for both SPCS 83 and SPCS2022)

#### *Defining values:*

- a semi-major axis = 6,378,137 meters (exact)
- f geometric flattening of the ellipsoid =  $1 / 298.257\ 222\ 101$  (derived quantity).  
More significant figures can be used (e.g.,  $1 / 298.257\ 222\ 100\ 882\ 711\ 243\dots$ ) but are not necessary; see NGS SPCS2022 Procedures, p. 10  
(<https://geodesy.noaa.gov/INFO/Policy/files/SPCS2022-Procedures.pdf>).

#### *Computed values (given to greater than double precision to provide a check):*

- b semi-minor axis =  $a(1 - f) = 6,356,752.314\ 140\ 347\ 438\dots$  meters
- e first eccentricity (not italicized) =  $\sqrt{f(2 - f)} = 0.081\ 819\ 191\ 042\ 831\ 850\ 707\dots$
- e<sup>2</sup> first eccentricity squared (not italicized) =  $f(2 - f) = 0.006\ 694\ 380\ 022\ 903\ 415\ 750\dots$

## F.1 - Lambert Conformal Conic Projection –Single Standard Parallel

### Notation and Definitions:

- $\varphi$  parallel of geodetic latitude **expressed as positive north values.**
- $\varphi_0$  central parallel, latitude of the true projection origin (*defining parameter*).
- $\lambda$  meridian of geodetic longitude **expressed as positive east values.**
- $\lambda_0$  central meridian, longitude of the true projection origin (*defining parameter*).
- $k$  grid scale factor at a general point.
- $k_0$  grid scale factor at the true projection origin ( $\varphi_0, \lambda_0$ ) (*defining parameter*).
- $\gamma$  convergence angle at a general point.
- $N$  northing (y) coordinate on the projection plane.
- $N_0$  false northing assigned to the true projection origin at ( $\varphi_0, \lambda_0$ ) (*defining parameter*).
- $E$  easting (x) coordinate on the projection plane.
- $E_0$  false easting assigned to the true projection origin at ( $\varphi_0, \lambda_0$ ) (*defining parameter*).
- $R$  mapping radius at latitude  $\varphi$ .
- $R_0$  mapping radius at  $\varphi_0$  (*projection specific constant*).
- $K$  mapping radius at equator (*projection specific constant*).
- $Q_\varphi$  isometric latitude for  $\varphi$  (see general function for Q below).

### **Projection Initialization:**

The general functions for the isometric latitude (Q) and a commonly occurring working term (W) associated with radius of curvature, both evaluated at a given latitude  $\varphi$ , are as follows:

$$Q_\varphi = \frac{1}{2} \left[ \ln \left( \frac{1 + \sin \varphi}{1 - \sin \varphi} \right) - e \ln \left( \frac{1 + e \sin \varphi}{1 - e \sin \varphi} \right) \right]$$

and

$$W_\varphi = \sqrt{1 - e^2 \sin^2 \varphi}$$

The following two zone specific constants are derived from the defining parameters  $k_0$  and  $\varphi_0$ :

$$R_0 = \frac{a k_0}{W_{\varphi_0} \tan \varphi_0}$$

$$K = R_0 \exp(Q_{\varphi_0} \sin \varphi_0)$$

where  $\exp(x) = e^x$  (*natural exponent*)

**Direct Conversion Computation ( $\varphi, \lambda$ )  $\rightarrow$  (N, E) with convergence ( $\gamma$ ) and scale factor (k):**

$$R_{\varphi} = \frac{K}{\exp(Q_{\varphi} \sin \varphi_0)}$$

$$\gamma = (\lambda - \lambda_0) \sin \varphi_0$$

$$N = N_0 + R_0 - R_{\varphi} \cos \gamma$$

$$E = E_0 + R_{\varphi} \sin \gamma$$

$$k = \frac{(W_{\varphi} R_{\varphi} \sin \varphi_0)}{a \cos \varphi}$$

**Inverse Conversion Computation (N, E)  $\rightarrow$  ( $\varphi, \lambda$ ) with convergence ( $\gamma$ ) and scale factor (k):**

$$R' = R_0 + N_0 - N$$

$$E' = E - E_0$$

$$R = \sqrt{R'^2 + E'^2}$$

$$Q' = \frac{\ln\left(\frac{K}{R}\right)}{\sin \varphi_0}$$

To compute  $\varphi$  start with the following initial approximation for  $\sin \varphi$ :

$$\sin_0 \varphi = \frac{\exp(2Q') - 1}{\exp(2Q') + 1}$$

Then iterate 4 times the following correction sequence for  $\sin_i \varphi$ :

$$c1_i = \frac{1}{2} \left[ \ln \left( \frac{1 + \sin_{i-1} \varphi}{1 - \sin_{i-1} \varphi} \right) - e \ln \left( \frac{1 + e \sin_{i-1} \varphi}{1 - e \sin_{i-1} \varphi} \right) \right] - Q'$$

$$c2_i = (1 - \sin_{i-1}^2 \varphi)^{-1} - e^2 (1 - e^2 \sin_{i-1}^2 \varphi)^{-1}$$

$$\sin_i \varphi = \sin_{i-1} \varphi - \frac{c1_i}{c2_i} \quad \text{for } 1 \leq i \leq 4$$

After which:

$$\varphi = \sin^{-1}(\sin_4 \varphi)$$

$$\gamma = \tan^{-1} \left( \frac{E'}{N'} \right)$$

$$\lambda = \lambda_0 + \frac{\gamma}{\sin \varphi_0}$$

$$k = \frac{W_{\varphi} R \sin \varphi_0}{a \cos \varphi}$$

## F.2 - Transverse Mercator Projection

### Notation and Definitions:

- $\varphi$  parallel of geodetic latitude **expressed as positive north values.**
- $\varphi_0$  latitude of the grid origin (*defining parameter*).
- $\lambda$  meridian of geodetic longitude **expressed as positive east values.**
- $\lambda_0$  central meridian (*defining parameter*).
- $\omega$  rectifying latitude.
- $k$  grid scale factor at a general point.
- $k_0$  grid scale factor assigned to the central meridian (*defining parameter*).
- $\gamma$  meridian convergence.
- $N$  northing (y) coordinate on the projection plane.
- $N_0$  false northing assigned to the grid origin at  $(\varphi_0, \lambda_0)$  (*defining parameter*).
- $E$  easting (x) coordinate on the projection plane.
- $E_0$  false easting assigned to the grid origin at  $(\varphi_0, \lambda_0)$  (*defining parameter*).
- $R$  radius of curvature in the prime vertical.
- $r_0$  geometric mean radius of curvature scaled to the grid.
- $r$  radius of the rectifying sphere (*projection specific constant*).
- $S$  meridional distance.
- $S_0$  meridional distance from the equator to  $\varphi_0$  multiplied by  $k_0$  (*projection specific constant*).

### **Projection Initialization:**

The general functions for the rectifying latitude ( $\omega$ ), a commonly occurring working term ( $W$ ) associated with radius of curvature, and a helper function ( $\eta_{\varphi}^2$ ), all evaluated at a given latitude  $\varphi$ , are as follows:

$$\omega_{\varphi} = \varphi + (\sin \varphi \cos \varphi) \{U_0 + \cos^2 \varphi [U_2 + \cos^2 \varphi (U_4 + U_6 \cos^2 \varphi)]\}$$

$$W_{\varphi} = \sqrt{1 - e^2 \sin^2 \varphi}$$

$$\eta_{\varphi}^2 = \frac{e^2 \cos^2 \varphi}{1 - e^2}$$

The following ellipsoid and projection specific constants are derived from relevant defining parameters and ellipsoid constants:

$$n = \frac{f}{2 - f}$$

$$r = a(1 - n)(1 - n^2) \left(1 + \frac{9n^2}{4} + \frac{225n^4}{16}\right) \quad 6,367,449.14577 \text{ meters GRS 80}$$

$$S_0 = r k_0 \omega_{\varphi_0}$$

$$u_2 = -\frac{3n}{2} + \frac{9n^3}{16} \quad u_4 = \frac{15n^2}{16} - \frac{15n^4}{32} \quad u_6 = -\frac{35n^3}{48} \quad u_8 = \frac{315n^4}{512}$$

$$\begin{array}{ll}
 U_0 = 2(u_2 - 2u_4 + 3u_6 - 4u_8) & -0.00504\ 82507\ 76226\ 4100\ \text{GRS}\ 80 \\
 U_2 = 8(u_4 - 4u_6 + 10u_8) & 0.00002\ 12592\ 04158\ 9984\ \text{GRS}\ 80 \\
 U_4 = 32(u_6 - 6u_8) & -0.00000\ 01114\ 23357\ 8320\ \text{GRS}\ 80 \\
 U_6 = 128u_8 & 0.00000\ 00006\ 26154\ 2943\ \text{GRS}\ 80
 \end{array}$$

$$v_2 = -\frac{3n}{2} - \frac{27n^3}{32} \quad v_4 = \frac{21n^2}{16} - \frac{55n^4}{32} \quad v_6 = \frac{151n^3}{96} \quad v_8 = \frac{1097n^4}{512}$$

$$\begin{array}{ll}
 V_0 = 2(v_2 - 2v_4 + 3v_6 - 4v_8) & 0.00502\ 28939\ 47825\ 8100\ \text{GRS}\ 80 \\
 V_2 = 8(v_4 - 4v_6 + 10v_8) & 0.00002\ 93706\ 25411\ 1503\ \text{GRS}\ 80 \\
 V_4 = 32(v_6 - 6v_8) & 0.00000\ 02350\ 59133\ 3050\ \text{GRS}\ 80 \\
 V_6 = 128v_8 & 0.00000\ 00021\ 80607\ 1774\ \text{GRS}\ 80
 \end{array}$$

**Direct Conversion Computation ( $\varphi, \lambda$ )  $\rightarrow$  (N, E) with convergence ( $\gamma$ ) and scale factor (k):**

$$t = \tan \varphi$$

$$L = (\lambda_0 - \lambda) \cos \varphi$$

$$S = r k_0 \omega_\varphi$$

$$R = \frac{a k_0}{W_\varphi}$$

$$A_2 = \frac{R t}{2}$$

$$A_4 = \frac{1}{12} [5 - t^2 + \eta_\varphi^2 (9 + 4 \eta_\varphi^2)]$$

$$A_6 = \frac{1}{360} [61 - t^2 (58 - t^2) + \eta_\varphi^2 (270 - 330 t^2)]$$

$$N = S - S_0 + N_0 + A_2 L^2 [1 + L^2 (A_4 + A_6 L^2)]$$

$$A_1 = -R$$

$$A_3 = \frac{1}{6} (1 - t^2 + \eta^2)$$

$$A_5 = \frac{1}{120} [5 - t^2 (18 - t^2) + \eta_\varphi^2 (14 - 58 t^2)]$$

$$A_7 = \frac{1}{5040} \{61 - t^2 [479 - t^2 (179 - t^2)]\}$$

$$E = E_0 + A_1 L \{1 + L^2 [A_3 + L^2 (A_5 + A_7 L^2)]\}$$

$$C_1 = -t$$

$$C_3 = \frac{1}{3} [1 + \eta_{\varphi}^2 (3 + 2 \eta_{\varphi}^2)]$$

$$C_5 = \frac{1}{15} (2 - L^2)$$

$$\gamma = C_1 L [1 + L^2 (C_3 + C_5 L^2)]$$

$$F_2 = \frac{1}{2} (1 + \eta_{\varphi}^2)$$

$$F_4 = \frac{1}{12} [5 - 4t^2 + \eta_{\varphi}^2 (9 - 24t^2)]$$

$$k = k_0 [1 + F_2 L^2 (1 + F_4 L^2)]$$

**Inverse Conversion Computation (N, E) → (φ, λ) with convergence (γ) and scale factor (k):**

$$\omega = \frac{N - N_0 + S_0}{k_0 r}$$

$$\varphi_f = \omega + (\sin \omega \cos \omega) \{V_0 + \cos^2 \omega [V_2 + \cos^2 \omega (V_4 + V_6 \cos^2 \omega)]\}$$

$$t_f = \tan \varphi_f$$

$$R_f = \frac{a k_0}{W_{\varphi_f}}$$

$$Q = \frac{E - E_0}{R_f} = \frac{(E - E_0) W_{\varphi_f}}{a k_0}$$

$$B_2 = -\frac{1}{2} t_f (1 + \eta_{\varphi_f}^2)$$

$$B_4 = -\frac{1}{12} \{5 + 3t_f^2 + \eta_{\varphi_f}^2 [(1 - 9t_f^2) - 4 \eta_{\varphi_f}^2]\}$$

$$B_6 = \frac{1}{360} \{61 + t_f^2 (90 + 45t_f^2) + \eta_{\varphi_f}^2 [46 - t_f^2 (252 + 90t_f^2)]\}$$

$$\varphi = \varphi_f + B_2 Q^2 [1 + Q^2 (B_4 + B_6 Q^2)]$$



$$B_3 = -\frac{1}{6}(1 + 2t_f^2 + \eta_{\varphi_f}^2)$$

$$B_5 = \frac{1}{120}[5 + t_f^2(28 + 24t_f^2) + \eta_{\varphi_f}^2(6 + 8t_f^2)]$$

$$B_7 = -\frac{1}{5040}\{61 + t_f^2[662 + t_f^2(1320 + 720t_f^2)]\}$$

$$L = Q\{1 + Q^2[B_3 + Q^2(B_5 + B_7Q^2)]\}$$

$$\lambda = \lambda_0 + \frac{L}{\cos \varphi_f}$$

$$D_1 = t_f$$

$$D_3 = -\frac{1}{3}[1 + t_f^2 - \eta_{\varphi_f}^2(1 + 2\eta_{\varphi_f}^2)]$$

$$D_5 = \frac{1}{15}[2 + t_f^2(5 + 3t_f^2)]$$

$$\gamma = D_1Q[1 + Q^2(D_3 + D_5Q^2)]$$

$$G_2 = \frac{1}{2}(1 + \eta_{\varphi_f}^2)$$

$$G_4 = \frac{1}{12}(1 + 5\eta_{\varphi_f}^2)$$

$$k = k_0[1 + G_2Q^2(1 + G_4Q^2)]$$

### F.3 - Oblique Mercator Projection – Natural Origin

#### **Notation and Definitions:**

- $\varphi$  parallel of geodetic latitude **expressed as positive north values.**
- $\lambda$  meridian of geodetic longitude **expressed as positive east values.**
- $\varphi_c$  central parallel, latitude of the local origin (*defining parameter*).
- $\lambda_c$  central meridian, longitude of the local origin (*defining parameter*).
- $N_c$  False Northing, assigned to the local origin ( $\varphi_c, \lambda_c$ ) (*defining parameter*).
- $E_c$  False Easting, assigned to the local origin ( $\varphi_c, \lambda_c$ ) (*defining parameter*).
- $k_c$  grid scale factor at the projection origin ( $\varphi_c, \lambda_c$ ) (*defining parameter*).
- $\alpha_c$  azimuth of positive skew axis (*u axis*) at the local origin ( $\varphi_c, \lambda_c$ ) (*defining parameter*).
- $\varphi'_0$  equator on aposphere – theoretical basis for the natural origin.
- $\lambda_0$  longitude of the natural origin.
- $\alpha_0$  azimuth of positive skew axis at the natural origin ( $\varphi'_0, \lambda_0$ )
- $N_0$  northing coordinate of the natural origin ( $\varphi'_0, \lambda_0$ ) – required to accommodate ( $N_c, E_c$ ).
- $E_0$  easting coordinate of the natural origin ( $\varphi'_0, \lambda_0$ ) – required to accommodate ( $N_c, E_c$ ).
- $Q_\varphi$  isometric latitude.
- $\chi_\varphi$  conformal latitude.
- $N$  northing (y) coordinate on the projection plane.
- $E$  easting (x) coordinate on the projection plane.
- $k$  grid scale factor at a general point.
- $\gamma$  convergence angle.

#### **Projection Initialization:**

The general functions for the isometric latitude (Q) and a commonly occurring working term (W) associated with radius of curvature, both evaluated at a given latitude  $\varphi$ , are as follows:

$$Q_\varphi = \frac{1}{2} \left[ \ln \left( \frac{1 + \sin \varphi}{1 - \sin \varphi} \right) - e \ln \left( \frac{1 + e \sin \varphi}{1 - e \sin \varphi} \right) \right]$$

and

$$W_\varphi = \sqrt{1 - e^2 \sin^2 \varphi}$$

The following procedure establishes ellipsoid specific constants for this projection type:

$$c_2 = \frac{e^2}{2} + \frac{5e^4}{24} + \frac{e^6}{12} + \frac{13e^8}{360}$$

$$c_4 = \frac{7e^4}{48} + \frac{29e^6}{240} + \frac{811e^8}{11520}$$

$$c_6 = \frac{7e^6}{120} + \frac{81e^8}{1120}$$

$$c_8 = \frac{4279e^8}{161280}$$

$$F_0 = 2(c_2 - 2c_4 + 3c_6 - 4c_8) \quad 0.00668 \ 69209 \ 27187 \ 3276 \text{ (GRS 80)}$$

$$F_2 = 8(c_4 - 34 + 10c_8) \quad 0.00005 \ 20145 \ 84388 \ 5346 \text{ (GRS 80)}$$

$$F_4 = 32(c_6 - 6c_8) \quad 0.00000 \ 05544 \ 29917 \ 8835 \text{ (GRS 80)}$$

$$F_6 = 128c_8 \quad 0.00000 \ 00068 \ 20452 \ 5428 \text{ (GRS 80)}$$

Projection specific constants are derived as follows (note: the equation for  $\lambda_0$  below has been modified from Stem to accommodate positive east longitude values):

$$A_1 = \frac{\sqrt{1 - e^2}}{W_{\varphi_C}}$$

$$A_2 = \frac{A_1}{W_{\varphi_C}}$$

$$B = \sqrt{1 + \frac{e^2 \cos^4 Q_{\varphi_C}}{1 - e^2}}$$

$$C = \cosh^{-1}\left(\frac{A_1 B}{\cos \varphi_C}\right) - B Q_{\varphi_C}$$

$$\alpha_0 = \sin^{-1}\left(\frac{\sin \alpha_C \cos \varphi_C}{A_1 B}\right)$$

$$\lambda_0 = \lambda_C - \frac{\sin^{-1}\left[\frac{\sin \alpha_0 \sinh(BQ_{\varphi_C} + C)}{\cos \alpha_0}\right]}{B}$$

$$D = a k_C A_2$$

$$I = B k_C A_2$$

To satisfy the condition that the false northing and false easting values be applied to the local origin  $(\varphi_C, \lambda_C)$  as opposed to the natural origin  $(\varphi'_0, \lambda_0)$ ,  $N_0$  and  $E_0$  are determined by applying a variant of the direct conversion procedure for  $(\varphi_C, \lambda_C)$  as follows:

$$L_C = (\lambda_0 - \lambda_C)B$$

$$J = \sinh(BQ_{\varphi_C} + C)$$

$$K = \cosh(BQ_{\varphi_C} + C)$$

$$v_1 = J \sin \alpha_0 + \cos \alpha_0 \sin L_C$$

$$u = D \tan^{-1} \left( \frac{J \cos \alpha_0 - \sin \alpha_0 \sin L_c}{\cos L_c} \right)$$

$$v = \frac{D}{2} \ln \left( \frac{K - v_1}{K + v_1} \right)$$

$$N_0 = N_c - u \cos \alpha_c + v \sin \alpha_c$$

$$E_0 = E_c - u \sin \alpha_c - v \cos \alpha_c$$

**Direct Conversion Computation ( $\varphi, \lambda$ )  $\rightarrow$  (N, E) with convergence ( $\gamma$ ) and scale factor (k):**

$$L = (\lambda_0 - \lambda) B$$

$$J = \sinh(BQ_{\varphi_c} + C)$$

$$K = \cosh(BQ_{\varphi_c} + C)$$

$$u_1 = \tan^{-1} \left( \frac{J \cos \alpha_0 - \sin \alpha_0 \sin L}{\cos L} \right)$$

$$v_1 = J \sin \alpha_0 + \cos \alpha_0 \sin L$$

$$u = D u_1$$

$$v = \frac{D}{2} \ln \left( \frac{K - v_1}{K + v_1} \right)$$

$$N = u \cos \alpha_c - v \sin \alpha_c + N_0$$

$$E = u \sin \alpha_c + v \cos \alpha_c + E_0$$

$$\gamma = \tan^{-1} \left( \frac{\sin \alpha_0 - A_4}{K \cos \alpha_0 \cos L} \right) - \alpha_c$$

$$k = \frac{I W_{\varphi} \cos u_1}{\cos \varphi \cos L}$$

**Inverse Conversion Computation (N, E)  $\rightarrow$  ( $\varphi, \lambda$ )**

Note: To compute convergence ( $\gamma$ ) and scale factor (k), apply resulting ( $\varphi, \lambda$ ) to the direct conversion equations above:

$$\Delta x = E - E_c$$

$$\Delta y = N - N_c$$

$$u = \Delta x \sin \alpha_c + \Delta y \cos \alpha_c$$

$$v = \Delta x \cos \alpha_c - \Delta y \sin \alpha_c$$

$$R = \sinh\left(\frac{v}{D}\right)$$

$$S = \cosh\left(\frac{v}{D}\right)$$

$$T = \sin\left(\frac{u}{D}\right)$$

$$q' = T \cos \alpha_0 - R \sin \alpha_0$$

$$Q' = \frac{\frac{1}{2} \ln\left(\frac{S+q'}{S-q'}\right) - C}{B}$$

$$\chi = 2 \tan^{-1} \left[ \frac{\exp(Q') - 1}{\exp(Q') + 1} \right]$$

$$\varphi = \chi + (\sin \chi \cos \chi) \{F_0 + \cos^2 \chi [F_2 + \cos^2 \chi (F_4 + F_6 \cos^2 \chi)]\}$$

$$\lambda = \lambda_0 + \frac{1}{B} \tan^{-1} \left[ \frac{R \cos \alpha_0 + T \sin \alpha_0}{\cos\left(\frac{u}{D}\right)} \right]$$